

# 國立高雄大學九十四學年度轉學招生考試試題

系所組別：應用數學系

科目：線性代數

考試時間：90 分鐘

本科原始成績滿分 100 分

滿分為100分。作答時請於答案卷上寫上題號及過程，只寫答案該題不予計分。

- Let  $T$  be the linear operator on  $P_2(\mathbb{R})$  defined by  $T(f(x)) = f(1) + f'(0)x + (f'(0) + f''(0))x^2$ , let  $\beta = \{1, x, x^2\}$ , and let  $A = [T]_{\beta}^{\beta}$ .
  - (5%) Find  $A$ .
  - (9%) Find the characteristic polynomial and all the eigenvalues of  $T$ .
  - (6%) For each eigenvalue  $\lambda$  of  $T$ , find the eigenspace  $E_{\lambda}$  corresponding to  $\lambda$ .
  - (5%) Find an invertible matrix  $Q$  and a diagonal matrix  $D$  such that  $AQ = QD$ .
  - (5%) Find  $A^{15}$ .
  - (5%) Find the minimal polynomial of  $A$ .
- (10%) Let  $A \in M_{n \times n}(\mathbb{C})$ . Prove that if  $A$  satisfies  $x^*Ax = 0$  for all  $x \in \mathbb{C}^n$ , then  $A = O_{n \times n}$ .
- (10%) Let  $A \in M_{n \times n}(\mathbb{C})$ . Prove that if  $A$  is positive definite, then  $e^A$  is positive definite.
- (10%) Let  $A \in M_{n \times n}(\mathbb{C})$ . Prove that
$$\dim(\text{span}(\{I_n, A, A^2, \dots\})) \leq n.$$
- (10%) Let  $A \in M_{m \times n}(\mathbb{C})$ . Prove that  $\text{rank}(A^*A) = \text{rank}(A)$ .
- (10%) Let  $A, B \in M_{n \times n}(\mathbb{C})$  be similar.
  - Prove that  $\det(A) = \det(B)$ .
  - Prove that  $\text{tr}(A) = \text{tr}(B)$ . [Note that  $\text{tr}(A)$  denoted by the sum of all the diagonal entries of  $A$ .]
- (15%) Define  $T : P(\mathbb{R}) \rightarrow P(\mathbb{R})$  by  $T(f(x)) = \int_0^x f(t) dt$ .
  - Prove that  $T$  is linear.
  - Prove that  $T$  is one-to-one.
  - Prove that  $T$  is not onto.

國立高雄大學九十四學年度轉學招生考試試題

系所組別： 應用數學系

科目： 微積分

考試時間： 90 分鐘

本科原始成績滿分 100 分

You need to show all your work.

1. (10 pts) Show that  $f(x) = \int_1^{2x} \sqrt{16 + t^4} dt$  has an inverse and find  $(f^{-1})'(0)$ .

2. Find the limit.

(a) (10 pts)  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{1/x^2}$

(b) (10 pts)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^9 y}{(x^6 + y^2)^2}$

3. (15 pts) For

$$g(x) = \begin{cases} x, & x \text{ rational,} \\ 0, & x \text{ irrational.} \end{cases}$$

Give an  $\epsilon, \delta$  proof for  $\lim_{x \rightarrow 0} g(x) = 0$ .

4. Find or evaluate the integral.

(a) (10 pts)  $\int \frac{1}{5 + 3 \sin x} dx$

(b) (10 pts)  $\int_0^1 x \ln x dx$

(c) (10 pts)  $\int_0^1 \int_0^{\cos^{-1} y} e^{\sin x} dx dy$

5. (15 pts) Find the area of the surface generated by revolving  $y = \cos x$ ,  $x \in [0, \frac{\pi}{2}]$  about the  $x$ -axis.

6. (10 pts) Find the values of  $p$  for which the series  $\sum_{n=2}^{\infty} \frac{\ln n}{n^p}$  converges.