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國立高雄大學九十五學年度轉學招生考試試題

科目:微積分

系所: 土木與環境工程學系二年級

考試時間:90分鐘

本科原始成績:滿分100分

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使用計算機

1. Find the first derivative of the function (10)

i
$$f(x) = \frac{x}{x + \frac{c}{x}}$$
, c is a constant

ii
$$g(\theta) = 5\theta \sec \theta + \theta \tan \theta$$

2. Find the limit (10)

$$i \sim \lim_{x \to 0} \frac{\sin x \left(1 - \cos x\right)}{4x^2}$$

ii
$$\lim_{x\to 0} \frac{1}{x^3} \int_0^x \sin(t^2) dt$$

- 3. Find an equation of the tangent line to $y^2 = 5x^4 x^2$ at the point (1, 2). (10)
- 4. If f is the focal length of a convex lens and an object is placed at a distance p from the lens, then its image will be at a distance q from the lens, where f, p and q are related by the lens equation

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Find the rate of change of p with respect to q. (10)

5. Evaluate the integral. (20)

$$i \cdot \int \frac{dx}{x \ln x}$$

ii
$$\int \cos\theta \cos^5(\sin\theta)d\theta$$

- 6. Find the arc length of the graph of $f(x) = \frac{1}{2}x^2$ from x = 0 to x = 1. (15)
- 7. The region R enclosed by the curves y = x and $y = x^2$ is rotated about the x-axis. Find the volume of the resulting solid. (10)
- 8. Find the center of mass of a solid hemisphere of radius R m if its density at height z m above the base plane of the hemisphere is $\delta_0 z$ kg/m³. (15)