

每題 10 分。

1. Solve $\frac{dy}{dx} = \frac{y^2 - 1}{x^2 - 1}$, $y(2) = 2$.

2. Solve $y''' - 2y'' + y' = 2 - 24e^x + 40e^{5x}$, $y(0) = \frac{1}{2}$, $y'(0) = \frac{5}{2}$, $y''(0) = -\frac{9}{2}$.

3. Solve $x^2y'' - 5xy' + 8y = 0$, $y(2) = 32$, $y'(2) = 0$.

4. Use the power series method to solve $(x-1)y'' + y' = 0$ about $x=0$.

5. Solve
$$\begin{aligned} \frac{dx}{dt} &= -4x + y + z \\ \frac{dy}{dt} &= x + 5y - z \\ \frac{dz}{dt} &= y - 3z. \end{aligned}$$

6. Find a basis for the nullspace of

$$A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

7. Let $T: R^2 \rightarrow R^3$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}$$

Find the matrix for the transformation T with respect to the bases $B = \{u_1, u_2\}$ for R^2 and $B' = \{v_1, v_2, v_3\}$ for R^3 , where

$$u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}; \quad v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

國立高雄大學九十五學年度轉學招生考試試題

科目：工程數學

系所：電機工程學系三年級

可

使用計算機

考試時間：90 分鐘

本科原始成績：滿分 100 分

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8. Find the orthogonal projection of the vector $u = [-3, -3, 8, 9]^T$ on the subspace of R^4 spanned by the vectors

$$u_1 = [3, 1, 0, 1]^T, \quad u_2 = [1, 2, 1, 1]^T, \quad u_3 = [-1, 0, 2, -1]^T.$$

9. Find the QR -decomposition of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

10. Find a matrix P that diagonalizes

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}.$$

- (10 分) Determine the current i in the circuit shown in Fig. 1.
- (15 分) Consider the linear time-invariant circuit shown in Fig. 2. At $t=0$ a constant voltage source of 10 volts is applied to the circuit. Find all branch voltages and all branch currents at $t=0$ and at $t=\infty$, given $i_L(0)=2$ amp and $v_C(0)=4$ volts.

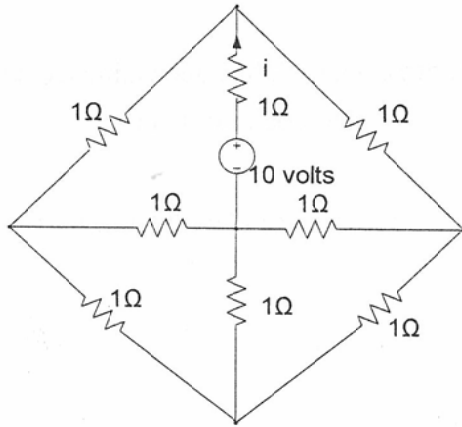


Fig. 1

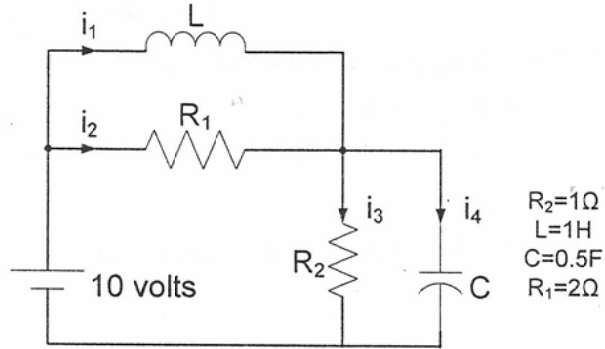


Fig. 2

- (15 分) Write the node equations for the linear time-invariant circuit shown in Fig. 3. Determine the differential equations for the voltages v_1 and v_2 . Indicate the needed initial conditions for each case in term of $v_C(0)$ and $i_L(0)$.
- (15 分) Repeat the preceding problem (see Fig. 3) using mesh analysis. (Hint: You have to transform the current source to an equivalent voltage source.)

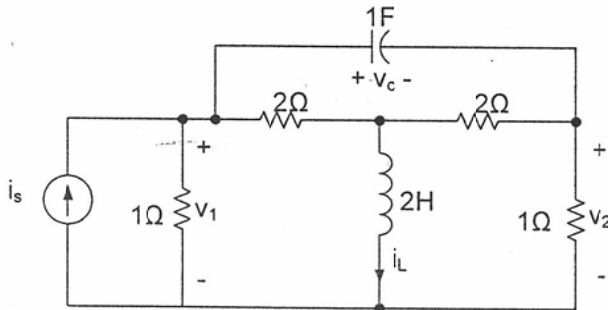


Fig. 3

- (10 分) Find the zero-input response [that is, $v_1(t)$ and $v_2(t)$, for $t \geq 0$] of the networks in Fig. 4.
- (15 分) Calculate the zero-state response of the networks in Fig. 4 when $e_1(t)=u(t)$, and $e_2(t)=\delta(t)$.

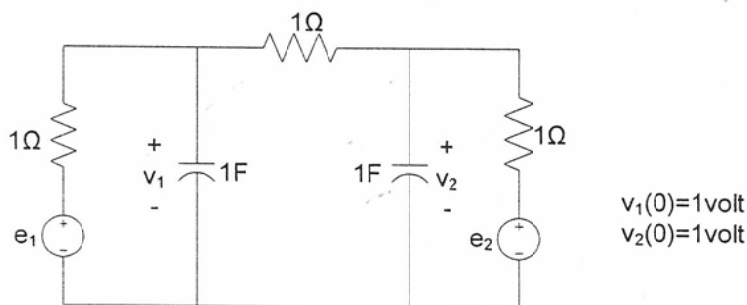


Fig. 4

7. (20 分) Two sets of measurements are made on a resistive network consisting of one known resistor and four unknown resistors, as indicated in Fig. 5. In the first measurement we have $i_1=0.6i_s$ and $i_1'=0.3i_s$, as shown in Fig. 5(a); in the second measurement, we have $i_2=0.2i_s$ and $i_2'=0.5i_s$, as shown in Fig. 5(b).

- Use the reciprocity theorem to calculate R_1 .
- Consider the configuration of sources shown in Fig. 5(c), where k has been adjusted so that no voltage appears across R_3 (that is, $i_3=i_3'$). Use the superposition theorem to determine this value of k .
- From the value of k obtained above, calculate i_3 ($=i_3'$) in terms of i_s , and hence determine R_2 and R_4 .
- Determine R_3 , using either measurement.

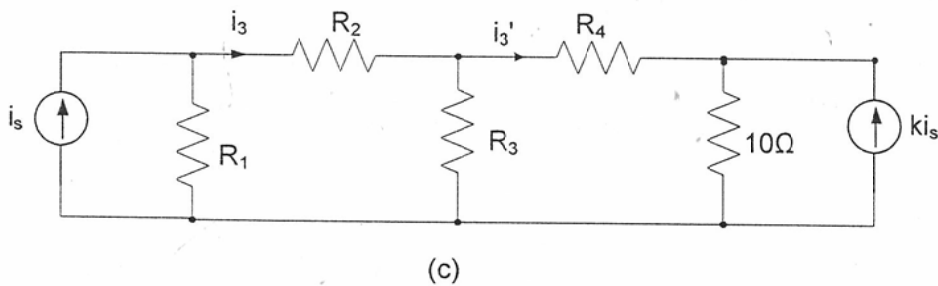
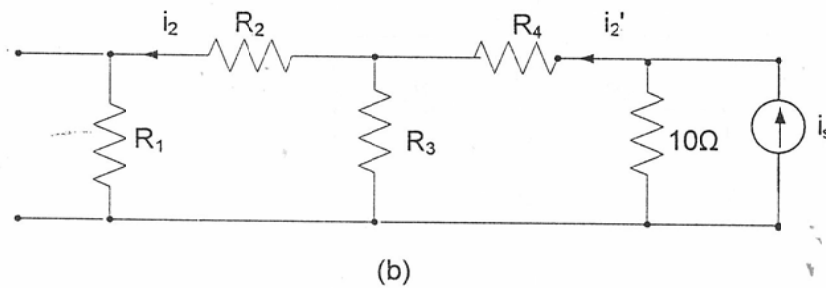
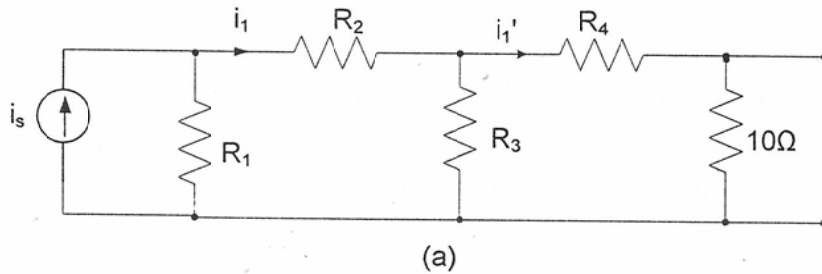


Fig. 5