國立高雄大學九十六學年度轉學招生考試試題

科目：微積分　　　　系所：應用數學系　　　　是否使用計算機：是
考試時間：90 分鐘　　本科原始成績：100 分

1. (10%) Show that the cubic polynomial $p(x) = x^3 + ax + b$ has exactly one zero for $a > 0$.

2. (10%) Find the constants a such that the function

$$f(x) = \begin{cases} \dfrac{4\sin x}{x}, & x > 0 \\ a - 2x, & x \le 0. \end{cases}$$

is continuous on $\mathbb{R}$.

3. (10%) Find equations for the tangent and normal lines of the equation $\tan xy = x$ at the point $\left(1, \frac{\pi}{4}\right)$.

4. (10%) Evaluate the integral $\int_{-2}^{4} f(x)\,dx$, where $f(x) = \begin{cases} x^2, & -2 \le x < 0 \\ xe^x, & 0 \le x \le 4. \end{cases}$.

5. (10%) Let f be continuous and define F by

$$F(x) = \int_0^x \left[t \int_1^t f(u)\,du \right] dt.$$

Find (a) $F'(x)$. (b) $F'(1)$. (c) $F''(x)$. (d) $F''(1)$.

6. (10%) Find the volume of the solid generated by revolving the region between $y = x^2$ and $y = 2x$ about the x–axis.

7. (10%) Find the limit (if it exists).

$$(a)\ \lim_{x\to 0^+} x^x, \quad (b)\ \lim_{x\to\infty} \left(\sqrt{x^2+2x} - x\right).$$

8. (10%) Let g be a twice-differentiable function of one variable and set

$$h(x, y) = g(x+y) + g(x-y).$$

Show that

$$\frac{\partial^2 h}{\partial x^2} = \frac{\partial^2 h}{\partial y^2}.$$

9. (10%) Find the absolute extreme values taken on by $f(x, y) = 2x^2 + y^2 - 4x - 2y + 2$ on the set $D = \{(x, y) : 0 \le x \le 2, 0 \le y \le 2x\}$.

10. (10%) Evaluate the double integral

$$\int_0^1 \int_y^1 e^{y/x}\,dx\,dy.$$

國立高雄大學九十六學年度轉學招生考試試題

科目：線性代數　　系所：應用數學系　　是否使用計算機：是
考試時間：90 分鐘　　本科原始成績：100 分

Notations:

$M_{m\times n}(\mathbb{R})$: the set of $m \times n$ matrices with entries in $\mathbb{R}$.
A^{-1}: the inverse of the matrix A.
$\det(A)$: the determinant of matrix A.
$\text{rank}(A)$: the rank of the matrix A.
$A^{\top}$: the transpose of matrix A.

1 (32) Determine each follow statement either is true or false. If true, prove it; if false, give a counterexample.

a If $A, B \in M_{n\times n}(\mathbb{R})$ are similar then they have the same eigenvalues.

b If $A \in M_{n\times n}(\mathbb{R})$ is nonzero matrix and v_1, v_2 are linearly independent then Av_1, Av_2 are linearly independent.

c If W_1, W_2 are subspaces of a vector space, then $W_1 \cap W_2$ is a subspace.

d For every matrix $A \in M_{n\times n}(\mathbb{R})$, there is a scalar $\lambda \in \mathbb{R}$ such that $A + \lambda I_n$ is NOT invertible.

2 (23) If $A = \begin{bmatrix} 7 & -2 & -4 \\ 2 & 1 & -2 \\ 2 & -1 & 0 \end{bmatrix}$ then

a. (8) Find the eigenvectors and eigenvalues of A.
b. (5) Find an invertible matrix C and a diagonal matrix D such that $A = CDC^{-1}$.
c. (5) Let $B = A^{21} + A^{11} + A$. Find the eigenvectors and eigenvalues of B.
d. (5) Find the eigenvectors and eigenvalues of A^{-1}.

3 (15) Let $W = \text{span}\{[2,1,3,4]^{\top}, [3,1,1,5]^{\top}\}$.
a. Find the orthogonal complement of W.
b. Find an orthonormal basis for W.
c. Find the projection of $[4,3,1,4]^{\top}$ on W.

4 (10) Find the value k that satisfies the following equation:

$$\det\begin{bmatrix} 3a_1 & 3a_2 & 3a_3 \\ 3b_1+5c_1 & 3b_2+5c_2 & 3b_3+5c_3 \\ 7c_1 & 7c_2 & 7c_3 \end{bmatrix} = k \det\begin{bmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

5 (10) Let $A \in M_{4\times 4}(\mathbb{R})$ and rank(A)=2, rank(B)=2
a. Find the matrices A and B, such that rank(AB)=1.
b. Find the matrices A and B, such that rank(AB)=0.

6 (10) Let $A \in M_{m\times n}(\mathbb{R})$. If $A^{\top}A = 0$ then $A = 0$.