國立高雄大學九十七學年度轉學招生考試試題

科目：微積分
考試時間：80 分鐘

系所：
應用數學系轉二年級
本科原始成績：100 分

是否使用計算機：是

1. (25%) Determine whether the statement is true or false. Please, explain why(If it is true, please show it. If it is false, given an example that shows it is false).
   (a) If f is undefined at $x = c$, then the limit of $f(x)$ as x approaches c does not exist.
   (b) If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$, then $\lim_{x\to\infty} [f(x) - g(x)] = 0$.
   (c) If $f(x) = g(x)$ for $x \neq c$ and $f(c) \neq g(c)$, then either f or g is not continuous at c.
   (d) If f is a bounded function on $[a, b]$, then f is an integrable function on $[a, b]$.
   (e) If $\lim_{(x,y)\to(0,0)} f(0, y) = 0$, then $\lim_{(x,y)\to(0,0)} f(x, y) = 0$.

2. (10%) Evaluate the definite integral:

$$(a)\ \int_0^4 |x^2 - 4x + 3|\, dx, \quad (b)\ \int_{-15}^{15} (x^{101} + x^{99} + \cdots + x^3 + x)\, dx.$$

3. (10%) Find the length of the curve $\overrightarrow{r}(t) = 3t\cos t\,\overrightarrow{i} + 3t\sin t\,\overrightarrow{j} + 4t\,\overrightarrow{k}$ from $t = 0$ to $t = 4$.

4. (10%) Find the two $x-$intercepts of

$$f(x) = x^2 - 3x + 2$$

   and show that $f'(x) = 0$ at some points between the two intercepts.

5. (10%) Find the line integral

$$\int_C F \cdot T ds$$

   where $F(x, y) = (x, xy)$, C is the unit circle $x^2 + y^2 = 1$ oriented in the clockwise direction and T is the unit tangent vector of C.

6. (10%) Show that the function

$$z = e^x \sin y$$

   satisfies Laplace's equation $\partial^2 z/\partial x^2 + \partial^2 z/\partial y^2 = 0$.

7. (15%) Let f be continuous on the interval $[0, b]$. Show that

$$\int_0^b \frac{f(x)}{f(x) + f(b - x)} dx = \frac{b}{2}.$$

   Use this result to evaluate

$$\int_0^1 \frac{\sin x}{\sin(1 - x) + \sin x} dx.$$

8. (10%) Find the equation of the tangent plane to the paraboloid

$$z = 1 - \frac{1}{10}\left(x^2 + 4y^2\right)$$

   at the point $\left(1, 1, \frac{1}{2}\right)$.

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科目：線性代數
考試時間：80 分鐘

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本科原始成績：100 分

是否使用計算機：是

Notations:

$N(T)$: the null space of T.
$R(A)$: the range of T.
$[T]_\beta^\gamma$: the matrix representation of T in bases β and γ.
P_k: the set of polynomials of degree at most k.

1 (24) Determine each follow statement either is true or false. If true, prove it; if false, give a counterexample.

a If A and B are similar square matrices and A is diagonalizable, then B is also diagonalizable.

b Every invertible matrix is diagonalizable.

c If $u+v$ lies in a subspace W of a vector space V, then both u and v lie in W.

2 Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be linear transformation and

$$\begin{aligned} T([0,0,1]^\top) &= [2,1,2]^\top, \\ T([1,1,1]^\top) &= [0,1,2]^\top, \\ T([-1,0,1]^\top) &= [1,0,0]^\top. \end{aligned}$$

a.(4) Find $T([0,10,10]^\top)$.
b.(8) Find $T([x,y,z]^\top)$.
c.(10) Find $N(T)$, $R(T)$, nullity(T) and rank(T).

3 Let $A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$.
a. (10) Find the eigenvalues and the corresponding eigenvectors of A.
b. (5) Compute A^k in terms of k.
c. (5) Find a polynomial $f(t)$ of degree 2, such that $f(A) = 0$.

4 Let $T : P_3 \to P_2$ be defined by $T(p(x)) = p'(2x+1)$, and let $\beta = \{x^3, x^2, x, 1\}$ and $\gamma = \{x^2, x, 1\}$ be the ordered bases for P_3 and P_2, respectively.
a.(10) Find the matrix representation $A = [T]_\beta^\gamma$ of T.
b.(4) Use A to compute $T(4x^3 - 5x^2 + 4x - 7)$.

5 (10) Let $W = \text{span}\{[1,0,1],[0,1,2],[2,1,4]\}$.
a. Find the orthogonal complement of W.
b. Find an orthonormal basis for W.

6 (10) Find a formula for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ that projects vectors on the plane $x+y+z=0$.