Production Outsourcing under Uncertainty

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Abstract
This paper is motivated by the observation that the nature of production outsourcing is sensitive to industry-specific capital investment under uncertainty. In particular, firms substantially involves in fixed-asset investment outsource at a greater level than those that engage less (even zero) in capital investment. We study an outsourcing model under uncertainty in which the firm’s attitudes towards risk being modeled by the assumption of mean-variance preferences, both with and without the assumption that outsourcing is available to a firm who incurs industry-specific capital investment under uncertainty.

1. Introduction
Recent development in the global semiconductor industry suggest two trends. First, ??????????. Second, ??????????
A main objective of this paper is to investigate a potential explanation for the nature of outsourcing. In particular, our theory determines the portfolio of self- and outsourcing-production that will emerge in equilibrium under uncertainty. The basic underlying idea is that a firm can employ sourcing as a risk-shifting device against demand uncertainty. This assumption allows us to investigate the hypothesis that the nature of outsourcing is sensitive to the fluctuation in demand. Furthermore, we explore the impact on outsourcing of industry-specific fixed-asset investment in an uncertainty setting, both with and without the assumption that outsourcing is available to any needed firms.

The purpose of this paper is to develop a model that is rich enough to capture some of the central features of a sourcing firm’s decision under uncertainty but

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simple enough to yield sharp insights into some of the central questions of capital investment and labor employment which that decision raises. The underlying theme is the comparison between fixed-asset investment with sourcing and that without under uncertainty. This is indeed a reflection of reality, at least, in the industries of semiconductor and sports-ware and has received much attention in the trade literature.

Section 2 describes the model. This is cast in the literally mean-variance preferences terms of outsourcing induced by fluctuations in demand on some final product but can be interpreted much more generally. Outsourcing is indeed the central concern behind the theme of global economic integration through trade but characterized by production disintegration (Feenstra, 1998; Helpman and Grossman, 2002) and has been extensively employed as an important corporate strategy (Shy and Stenbacka, 2003) in the global semiconductor industry, for example, about 87% of the wafer manufacturing was outsourced to Taiwan (Fabless Association, 2003). Also central to the model is an assumption that outsourcing is definitely available at all time, namely, the probability of sourcing firm matching with a subcontractor who impeccably delivers the sourcing order equals one. Thus, the analysis can thus be viewed either as providing an individual-choice perspective on strategic aspects of outsourcing in an uncertainty context or - our preferred interpretation - as a groundwork for the conventional welfarist treatment of outsourcing issue for the case in which firms gain from subcontracting.

In Section ??, we derive the optimal portfolios of sourcing- and self-production in a two-period setting an on the assumption that the firm engages in fixed-asset investment. Our main results here support ??????????. We show that ???????????????. Our theory, therefore, can explain ???????????????.

We conclude in Section ?? with some final remarks.

2. The basic model

3. Outsourcing under uncertainty with no fixed asset investment

Here we examine its choices of outsourcing production in equilibrium when the sourcing firm involves no fixed-asset investment even under uncertainty. More precisely, we characterized the equilibrium portfolio when a sourcing firm with mean-variance preferences behaves ranks the profit streams solely by the average properties captured in its mean and variance.
The first task is to characterize the sourcing firm’s expected profit. This we do from the perspective of information structure. Thus we ask: given \( X \) (which is not known until the beginning of time 1), what \( Y, y \) even both maximize the sourcing firm’s expected profit, bearing in mind the mean-variance profit function?

We start by considering that outsourcing is not possible, so that all output demand must be met by the firm’s self-production, namely, \( y = 0 \). Hence the firm’s problem is essentially choosing a level of its own production to maximize the expected profit

\[
\max_Y (\bar{X} \cdot Y^{1-\varepsilon} - MY^{\lambda_s}).
\]

Using the first order conditions of (1), we get the following proposition.

**Proposition 1 (Existence of the equilibrium).** If \( \varepsilon + \lambda_s > 1 \), there exists a unique solution \( Y^n \in (0, \infty) \) to (1).\(^3\) Moreover, if the expectation of the demand uncertainty \( \bar{X} \) is sufficiently high, then \( Y^n \) becomes sufficiently large.

On the other hand, when outsourcing is possible, i.e., \( y > 0 \), at time 1, the firm chooses both outputs of the self-production \( Y \) and the outsourcing-production \( y \) to maximize its expected profit of

\[
\max_{Y,y} (\bar{X}(Y + y)^{1-\varepsilon} - MY^{\lambda_s} - (C + M y^{\lambda_o})).
\]

The first order condition yields

\[
\begin{align*}
\bar{X}(1 - \varepsilon)(Y + y)^{-\varepsilon} &= M\lambda_s Y^{\lambda_s - 1}, \\
\bar{X}(1 - \varepsilon)(Y + y)^{-\varepsilon} &= M\lambda_s y^{\lambda_o - 1},
\end{align*}
\]

**Proposition 2 (Existence of the equilibrium).** If \( \varepsilon + \lambda_s > 1 \) and \( \varepsilon + \lambda_o > 1 \), then the optimization problem (2) has a unique solution of \( (Y^w, y^w) \), where \( Y^w \) and \( y^w \) satisfy

\[
Y^w = \left( \frac{\lambda_o}{\lambda_s (y^w)^{\lambda_o - 1}} \right)^{\frac{1}{\lambda_s - 1}}.
\]

**Lemma 1.** If \( y^w > 0 \), then both \( Y^w \) and \( y^w \) rise with a high value of \( \bar{X} \).

**Proposition 3.** (1) The level of self-production output under no outsourcing is greater than with outsourcing possible, i.e., \( Y^n > Y^w \).

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\(^3\)The condition \( \varepsilon + \lambda_s > 1 \) is necessary. If \( \varepsilon + \lambda_s < 1 \), the optimization problem max \( Y E[V(Y,0)] \) has no maximum, since \( \lim_{Y \to \infty} E[V(Y,0)] = \infty \). If \( \varepsilon + \lambda_s = 1 \), we have to separate it into two cases: If \( \bar{X} > M_s, E[V(Y,0)] \) has no maximum. If \( \bar{X} < M_s, E[V(Y,0)] \) has a maximum at \( Y = 0 \).
(2) The level of total production under no outsourcing is lower than that with outsourcing, i.e., \( Y^n > Y^w + y^w \).

Hence, throughout of the rest of this paper we assume that \( \varepsilon + \lambda_o > 1 \) and \( \varepsilon + \lambda_o > 1 \).

Proof of Lemma 1. We see that if the constant \( \bar{X} \) is smaller, then the graph of the function \( h(y) \) shift parallel downward. Hence the solution to \( h(y) = 0, Y^w_o \) increases. Furthermore, from (4) we see also that \( Y^w_o \) is increasing. \( \square \)

Hence, if the demand of this production increases, then the firm should increase both its own production and the amount of outsourcing production.

The result contained in Proposition 2 suggests that the sourcing firm’s total output drops, however, its total expected profit rises if it decides to take on production outsourcing. The reason for this phenomena is that although the decrease of the total output reduces the firm’s revenue, the firm saves more money from the production costs due to the outsourcing. Furthermore, since the self-producing firm decides if he goes outsourcing at time 0, he has no enough information, since the firm has to take a conservative strategy and not to invest too much.

4. Mean-variance preference

First we define the mean-variance preference.

The property (C1) means that using strategy \((y_1, y_2)\) the expected profit of the product is larger than that of strategy \((\tilde{y}_1, \tilde{y}_2)\). The property (C2) means that the risk of using strategy \((y_1, y_2)\) is lower than that of using \((\tilde{y}_1, \tilde{y}_2)\). In this setting, we have seen that

\[
(5) \quad \text{Var}(V(Y_s, Y_o)) = \text{Var}(X)(Y_s + Y_o)^{2(1-\varepsilon)}.
\]

This means that the firm can minimize his risk only if the total production amount \( Y + y = 0 \), i.e., produces nothing at all. However, this means also that the firm has no any profit. Hence, we would like to find a strategy \((Y^*, y^*)\) which is no worse than the other strategies.

Proposition 4. For every \((Y, y) \in \mathbb{R}^+ \times \mathbb{R}^+\),

1. if \( Y < Y^n, (Y, 0) \not\preceq (Y^n, 0) \);
2. if \( Y > Y^n, (Y, 0) \preceq (Y^n, 0) \);
3. if \( Y + y < Y^w + y^w, (Y^w, y^w) \not\preceq (Y, y) \);
4. if \( Y^w + y^w < Y + y, (Y, y) \preceq (Y^w, y^w) \).
Proposition 5. Production with outsourcing is preferred to without outsourcing, i.e.,

$$(Y^n, 0) \preceq (Y^w, y^w).$$

That is, if the firm does go outsourcing, it increases the expected profit and decreases the risk.

From Proposition 5, we get that the expected profit in the case without outsourcing is smaller than that in the case with outsourcing. However, we also see that the expected revenue in the case without outsourcing is also smaller than that in the case with outsourcing. From this we see that the cost function in the case without outsourcing is also smaller than that in the case with outsourcing. It means that a self-producing firm works with an outsourcing firm can reduce his revenue function, but also reduce his cost. Moreover, the money he saves from the reduced cost is more than from the lost of revenue. Hence, the self-producing firm can increase his profit when he works with outsourcing firm.

Proof of Proposition 1

From the first order condition, we see that the optimal solution to (1) satisfies the equation

$$(6) \quad M_s \lambda_y y^\varepsilon + \lambda_s y^{-1} = \bar{X}(1 - \varepsilon).$$

Hence

$$(7) \quad Y^n = \left( \frac{\bar{X}(1 - \varepsilon)}{M_s \lambda_s} \right)^{1/(\lambda_s + \varepsilon - 1)}.$$ 

It is easy to check that $(Y^n, 0)$ is a (global) maximum of $E[V(Y, 0)]$ due to the second order condition. Furthermore, due to (7), we see that $Y^n_s$ is increasing in $\bar{X}$.

Proof of Lemma 2

From (3) we see that if $(Y^w, y^w)$ is a solution to (3), it should satisfy (4). It remains to check that $(Y^w, y^w)$ exists.

Consider the function

$$h(y) = (C + M_o \lambda_o y^{\lambda_o - 1}) \left( \left( \frac{1}{M_s \lambda_s} (C + M_o \lambda_o y^{\lambda_o - 1}) \right)^{\frac{1}{\lambda_s - 1}} + y \right)^{\varepsilon} - \bar{X}(1 - \varepsilon).$$

We see that this function $h$ is strictly increasing provided $\varepsilon + \lambda_s > 1$ and $\varepsilon + \lambda_o > 1$. Since $h$ is continuous, $h(0) = -\bar{X}(1 - \varepsilon) < 0$, $\lim_{y \to \infty} h(y) = \infty$, we get that the equation $h(y) = 0$ has a unique solution, say $y^w$. Substituting it into (4) we get $Y^w$.

Proof of Proposition 3
(1) We know that $Y^n$ and $Y^w$ satisfy

$$
M_s \lambda_s(Y^n)^{\lambda_s + \varepsilon - 1} = \bar{X}(1 - \varepsilon),
$$

(9)

$$
M_s \lambda_s(Y^w)^{\lambda_s - 1}(Y^w + y^w)^\varepsilon = \bar{X}(1 - \varepsilon).
$$

Define the function $f(y)$ by $M_s \lambda_s y^{\lambda_s + \varepsilon - 1}$. Then due to (9) and $Y^w, y^w > 0$, we get

$$
f(Y^w) = M_s \lambda_s (Y^w)^{\lambda_s + \varepsilon - 1} = \bar{X}(1 - \varepsilon) \left( \frac{Y^w}{Y^w + y^w} \right)^\varepsilon < \bar{X}(1 - \varepsilon) = f(Y^n).
$$

Since $f(y)$ is increasing in $y$, we obtain $Y^w < Y^n$.

(2) From (8) and (9) we get that

$$
M_s \lambda_s (Y^n)^{\lambda_s + \varepsilon - 1} = M_s \lambda_s (Y^w)^{\lambda_s - 1}(Y^w + y^w)^\varepsilon.
$$

Since $Y^w \leq Y^n$ by part (1) and $\lambda_s, \lambda_o < 1$, we get the desired result.

**Proof of Proposition 4**

From the construction we see that $(Y^n, 0)$ and $(Y^w, y^w)$ maximize the expectation of $E[V(Y, 0)]$ and $E[V(Y, y)]$, respectively. Hence, $E[V(Y^n, 0)] \geq E[V(Y, 0)]$ and $E[(Y^w, y^w)] \geq E[V(Y, y)]$ for all $Y \geq 0$ and $y \geq 0$. Thus, $(Y^n, 0) \not\leq (Y, 0)$ and $(Y^w, y^w) \not\leq (Y, y)$. For $Y > Y^n$, it's clearly that $\text{Var}(V(Y, 0)) > \text{Var}(V(Y^n, 0))$ which implies that $(Y, 0) \not\leq (Y^n, 0)$. Similarly, for $Y + y > Y^w + y^w$, we have $(Y, y) \not\leq (Y^w, y^w)$.

**Proof of Proposition 5**

(1) Since

$$
E[V(Y^w, y^w)] = \max_{Y, y} (\bar{X}(Y + y)^{1-\varepsilon} - C_s(Y) - C_o(y))
$$

$$
\geq \max_Y (\bar{X}(Y + 0)^{1-\varepsilon} - C_s(Y) - C_o(0))
$$

$$
= \max_Y (\bar{X}Y^{1-\varepsilon} - C_s(Y)) = E[V(Y^n, 0)].
$$

We obtain the first property in Definition ??.

(2) Since

$$
\text{Var}(V(Y, y)) = \text{Var}(X)(Y + y)^{2(1-\varepsilon)},
$$

we get $\text{Var}(V(Y^n, 0)) \geq \text{Var}(V(Y^w, y^w))$ due to Proposition 3 (2). We get the desired result.

**References**


