Note: You may not be able to find the exact probabilities for some problems. Try to explain your steps as clear as possible.

1. (20 points) For a simple regression model,

\[ Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \quad i = 1 \ldots n. \]

where \( Y_i \) and \( X_i \) are variables randomly drawn from a population; \( \epsilon_i \) is an error term and is not correlated with \( X_i \).

(a) (10 points) Assume that \( X_i \) is nonstochastic and \( \epsilon_i \) is i.i.d. \( N(0, \sigma^2) \). If \( \hat{\beta}_1 \) is the least squared estimators of \( \beta_1 \). Please find the exact distribution of \( \hat{\beta}_1 \).

(b) (10 points) Show that \( \hat{\beta}_1 \) are unbiased and it is the most efficient estimator among unbiased and linear estimators.

2. (15 points) Consider the linear probability model, \( Y_i = \beta_0 + \beta_2 X_i + \epsilon_i \), where \( Y_i \) is a binary variable equal to 1 if success and 0 if fail and \( \Pr(Y_i = 1|X_i) = \beta_0 + \beta_2 X_i \).

(a) (7 points) Please show that show that \( \epsilon_i \) is herteroskedastic and how to fix herteroskedastic problem. What else weakness does the linear probability model have?

(b) (8 points) Instead of modeling this problem by using linear probability, we suppose that \( \Pr(Y_i = 1|X_i) = \phi(\beta_0 + \beta_2 X_i) \), where \( \phi(\cdot) \) is a logistic c.d.f. function. In order to estimate \( \beta_0, \beta_2 \), the method of least square is no longer valid and Why? So, the method of Maximum Likelihood is usually applied here. Please write down the log of the likelihood function and briefly explain how to find the Maximum Likelihood Estimators of \( \beta_0, \beta_2 \).

3. (20 points) Let \( X_1 \) and \( X_2 \) have independent gamma distributions with parameters \( \alpha, \theta \) and \( \beta, \theta \), respectively. Let \( W = \frac{X_1}{X_2} + X_2 \). Find the p.d.f. of \( W \). What is the distribution of \( W \)?
4. (15 points) Earnings functions attempt to predict the log of earnings ($\gamma_i$) from a set of explanatory variables, both binary and continuous. You have allowed for an interaction between two continuous variables: years of education and tenure with the current employer. Your estimated regression is of the following type:

$$\tilde{Y}_i = \beta_0 + \beta_1 \text{Femme}_i + \beta_2 \text{Educ}_i + \beta_3 \text{Tenure}_i + \beta_4 \text{Educ}_i \cdot \text{Tenure}_i$$

where Femme is a binary variable taking on the value of one for females and is zero otherwise, Educ is the number of years of education, and Tenure is continuous years of work with the current employer.

(a) (5 points) What is the effect of an additional year of education on earnings (“returns to education”) for men? For women?

(b) (5 points) If you allowed for the returns to education to differ for males and females, how would you respecify the above regression?

(c) (5 points) What is the effect of an additional year of tenure with a current employer on earnings?

5. (10 pts) Bags of a chemical produced by a company have impurity weights that can be represented by a normal distribution with mean 12.2 grams and standard deviation 2.8 grams. A random sample of 400 of these bags is taken. What is the probability that at least 100 of them contain less than 10 grams of impurities?

6. Consider a fishing company on the coast of New England. It operates a search plane to find schools of salmon that are randomly located in the North Atlantic. On average, 1 school of salmon appears per 100,000 square miles of sea. On a given day, the plane can fly 1,000 miles, effectively searching a lateral distance of 5 miles on either side of its path.

(a) (10 pts) What is the probability of finding at least one school of salmon during 3 days of searching?

(b) (10 pts) How many days of search are needed before the probability of finding at least 1 school of salmon reaches 0.95?
Explain and show how a firm that follows the law of diminishing marginal returns can adjust its production scale in the long run.

Explain and show why a monopolistic firm produces an output at which demand is inelastic in the long run.

Suppose there are two markets, A and B. (15%)
(1) Suppose the total demand for a good is given as \( P = 80 - Q \) and the total supply is given as \( P = 2Q \). What is the price elasticity of demand, when the market is clearing?

(2) Suppose the demand for the good in market A is given as \( P = 80 - 4Q \). The good is provided by a monopolist that has two identical factories, \( \alpha \) and \( \beta \), in markets A and B respectively. What are the market prices and quantities provided by each factory, when two markets are perfectly isolated? What are the market prices and quantities provided by each factory, when freight is costless?

Suppose there are two markets, A and B. (15%)
(1) Suppose the total demand for a good is given as \( P = 80 - Q \) and the total supply is given as \( P = 2Q \). What is the price elasticity of demand, when the market is clearing?

(2) Suppose the demand for the good in market A is given as \( P = 80 - 4Q \). The good is provided by a monopolist that has two identical factories, \( \alpha \) and \( \beta \), in markets A and B respectively. What are the market prices and quantities provided by each factory, when two markets are perfectly isolated? What are the market prices and quantities provided by each factory, when freight is costless?

(3) Suppose the demand for a good is given as \( P = 80 - Q \), the total supply is given as \( P = 2Q \). When the market is clearing, what is the price elasticity of demand?

Given the market demand function for a product \( P(Q) = 50 - 2Q \), where \( Q = q_1 + q_\beta \cdot q_\beta \), with \( q_1 \) and \( q_\beta \) being the quantities produced by firms 1 and 2, respectively. Let the cost function be \( C(q) = 2q_i \cdot i = 1, 2 \). (20%)

(1) Consider two firms \( \text{Cournot} \) competition. When both firms produce the same quantity, calculate the total profit for each firm.

(2) The leader in the market produces \( q_1 \) for each firm, the follower produces \( q_\beta \) for each firm. Calculate \( \text{Stackelberg} \) equilibrium for each firm.

(3) Calculate the profits in \( \text{Cournot} \) and \( \text{Stackelberg} \) equilibrium.
1. (25%)  
   (a) What is “sterilization” policy of the central bank? (5%)  
   (b) Is it possible for a nation’s central bank to implement a full sterilization operation while its capital market is perfectly mobile? Why? (5%)  
   (c) For a country of large degree of capital control with fixed exchange rate regime, use Fleming (1962) framework of full sterilization to compare the effect of expansionary monetary policy on output for small and large elasticities of domestic investment. (15%)  

2. (25%) The figure below shows the inflation and unemployment in the US since 1961.  
   (a) What is “Phillips curve”? (5%)  
   (b) Is there any evidence showing the existence of Phillips curve in the US since 1961? Please write down the Phillips curve equation and conclude your findings. (20%)  

![Inflation vs Unemployment](image)  

3. (25%) Please use Solow’s neoclassical growth model to answer the following questions.  
   (a) What is “steady state”? (5%)  
   (b) What is “gold rule” allocation? (5%)  
   (c) Use this framework to analyze the short- and long-run effects on consumption, saving and output of a shock, like Rebar group scandal, causing the capital stock dropped suddenly from its original level at steady state. (15%)  

4. (25%)  
   (a) What is “Ricardian equivalence”? (5%)  
   (b) Please state the main differences between Keynes’ and Friedman’s consumption function. (10%)  
   (c) Compare the effect on current consumption and saving of a temporarily dropped (starting from next year and lasting for 6 months) in income tax. (10%)