1. Let \( \texttt{a[]} = \{3, 7, 13, 47, 51, 58, 62, 77, 91\} \) be an integer array in C. How many numbers would be checked if we search a key 58 in \( \texttt{a[]} \) with (a) linear search and (b) binary search, respectively? (10%)

2. (a) Represent decimal number -19 using 8-bit 2's complement representation. (5%) (b) Show how to compute 3 - 18 using 2's complementation representations and an addition (each number is represented by 8 bits). (5%)

3. All the following questions refer to C++ language. (a) Destructor functions are not needed for some classes you define but are essential to some others. Please give an example for which destructor function is a need. (5%) (b) Assignment operator ("=") can be used to copy contents from one object to another when these two objects are of the same class. However, sometimes you need to define an overloaded assignment operator even for objects of the same type. Please give an example to explain when and why. (5%)

4. The following is a prototype of a C function:

   ```c
   void permutation(int n[], int low, int high);  
   ```

   This function prints all possible combinations of 0's and 1's for bits ranged from \( n[\text{low}] \) to \( n[\text{high}] \). For example, it prints 0000, 0001, ..., 1111 for the following main program:

   ```c
   int main()  
   {  
      int n[4];  
      
      permutation(n, 0, 3);  
      return 0;  
   }
   ```

   You are required to design `permutation` as a recursive function. You may write down the code in C or C++. (10%)

5. A half adder has four bits: A, B, Sum, and Carry. (a) Show the truth table of a half adder. (5%) (b) Draw the circuit diagram of a half adder. (5%)

6. Justify the correctness of the following statements. (5%)
   (a) \( \log n \in O(n) \)
   (b) \( n \in O(n \log n) \)
(c) \( n \log n \in O(n^2) \)
(d) \( 2^n \in W(5^n) \)
(e) \( n^{1.5} \in W(n \log n) \)

7. Consider the following circular linked list pointed by pointer list.

```
1 --> 2 --> 3 --> 4
```

Let \( p \) be a pointer to a node. The following notation is used.
- \( \text{info}(p) \): the information field of a node
- \( \text{next}(p) \): the pointer field to the next node

Please write the steps to invert this linked list into the following result. (10%)

```
4 --> 3 --> 2 --> 1
```

8. Please explain how to utilize the binary search tree structure to devise a sorting algorithm. What is the computing time complexity of your algorithm? (10%)

9. Consider the following directed graph.

(a) Give a topological ordering of this graph. (5%)
(b) Devise an algorithm with complexity of \( O(n + e) \) to find a topological order, where \( n \) and \( e \) denote the number of vertices and edges, respectively. (10%)

![Directed Graph]

10. A double ended queue (deque) is a linear list in which insertions and deletions may be made at either end. Consider an integer sequence: 1 2 3 4. Could any permutation of 1 2 3 4 be obtained by first inserting this sequence into a deque and then outputting all of the values? Please give your reason. (10%)
1. (a) (5%) \( S = \{1, 2, 3, 4, 5, 6, 7, 8\} \). Show that if any 6 integers are selected from \( S \), there are at least two whose sum is 8.

(b) (5%) Let \( \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\} \subseteq \mathbb{Z}^+ \). Show that for some \( i \neq j \), either \( x_i + x_j \) or \( x_i - x_j \) is divisible by 8.

2. (a) (7%) Five distinct toys are distributed to five children (one toy to each child). In how many ways can these five distinct toys be collected and redistributed to the five children so that each child gets one different toy from the first distribution?

(b) (8%) Five married couples are to be seated at a circular table. In how many ways can they arrange themselves so that no wife sits next to her husband?

3. (10%) For all \( n \in \mathbb{Z}^+ \), show that \( n^2 + (n+1)^2 + (n+2)^2 + (n+3)^2 \) is not divisible by 8.

4. Explain why it is not possible to draw a loop-free connected undirected graph with 10 vertices, where the degrees of the vertices are
   (a) (5%) 2, 2, 2, 3, 4, 5, 7, 9 or
   (b) (5%) 1, 1, 1, 3, 3, 4, 4, 5, 7, 9.

5. (10%) For \( n \in \mathbb{Z}^+ \), let \( D_n \) be the following \( n \times n \) determinant:

\[
\begin{vmatrix}
1 & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\
-1 & 1 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & -1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & -1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & -1 & 1 \\
\end{vmatrix}
\]

Find and solve a recurrence relation for the value \( D_n \).

6. For each \( n \in \mathbb{Z}^+ \), show that
   (a) (5%) \( \gcd(n, n+1) = 1 \).
   (b) (5%) The number of positive divisors is less than or equal to \( 2\sqrt{n} \).
7. (a) (5%) For every positive integer \( m, n \), with \( m < n \), show that

\[
\sum_{k=0}^{n} (-1)^k \binom{n}{k} (n-k)^m = 0
\]

(b) (5%) There are 8 people and four kinds of balls which are blue, red, green, and white. In how many ways can each person select a ball so that each color is selected at least once?

(c) (5%) Answer part (b), assuming John, among these 8 people, selects a blue ball.

8. (a) (5%) Find the coefficient of \( x^{20} \) in \( (x + x^2 + x^3 + x^4 + x^5 + x^6)^{10} \).

(b) (5%) If a fair die is rolled 10 times, what is the probability that the sum of the rolls is 30?

9. If \( G = (V, E) \) is an undirected graph, a proper coloring of \( G \) occurs when we color the vertices of \( G \) so that if \( \{a, b\} \) is an edge in \( G \), then \( a \) and \( b \) are colored with different colors.

The minimum number of colors needed to properly color \( G \) is written \( \chi(G) \). For the following graph,

(a) (5%) Find \( \chi(G) \).

(b) (5%) If five colors are available, in how many ways can the vertices be properly colored?
1. (10%) Given a computer system with a 32-bit virtual address, 4KB pages, and 8 bytes per page entry, suppose that the maximum physical memory size is 64GB, and the system is byte-addressable. Let paging be implemented for the system. Please answer the following questions:
   a. What is the number of bits for physical addresses? What is the maximum number of frames for the system? What is the maximum number of pages for a process?
   b. Suppose that multi-level paging is adopted! How many levels do we have? Let the memory access time and TLB access time be 100ns and 20ns, respectively. Suppose that the TLB hit ratio is 99%. What is the effective memory access time?
   c. Suppose that the virtual memory of the computer system adopts demand paging. Suppose that the effective memory access time of the computer system without any page fault be 100ns, and the service time for a page fault be 15ms. If the page fault rate is 0.0000004, what is the effective access time under demand paging?

2. (10%) Explain the main difference between a process and a thread? Please compare user-level threads and kernel-level threads. Explain under what circumstances is one type better than the other.

3. (10%) Please define race condition. Please compare the difference between busy waiting and blocking.

4. (10%) I/O could be done in various ways. The major types are synchronous and asynchronous I/O's.
   a. Please define synchronous and asynchronous I/O's. Please also compare these two kinds of I/O.
   b. Could DMA be used with synchronous I/O? You must provide explanation.

5. (10%) Please compare CPU scheduling algorithms FCFS and Shortest Job First (SJF).

6. (10%) Describe data migration, computation migration, and process migration in a distributed system.
7. (10%) What is dual-mode operation? Why an O.S. needs to implement memory/CPU protection? Explain how to implement the protection scheme.

8. (8%) What is system call? Please describe how an O.S. manages a system call.

9. (10%) There are 4 necessary conditions for a deadlock. What are they? What is the logic that can be used these conditions to develop deadlock free algorithms? Please compare the deadlock prevention and deadlock avoidance algorithms?

10. (12%) Suppose the head of a moving-head disk with 3000 tracks, numbered 0 and 2999, has just finished serving a request at track 76. If the queue of pending requests is kept in the FIFO order: 45, 725, 475, 887, 469, 754, 511, 875, 73. Starting from the current position, what is the total distance (in tracks) that the disk arm moves to satisfy these requests for the following disk scheduling algorithms?
   (a) FCFS
   (b) SSTF
   (c) SCAN
   (d) C-SCAN
   (e) LOOK
   (f) C-LOOK