1. True/False (3% for each; total 30%)
   (1) Let \( m, n \in \mathbb{Z} \). Then \( n \cdot C(m+n, m) = (m+1) \cdot C(m+n, m+1) \)
   (2) \( +5 \) is a positive integer. \( ? \) is a statement.
   (3) \( \forall x \exists y \ [p(x, y) \rightarrow q(x, y)] \leftrightarrow \exists y \forall x \ [\neg p(x, y) \lor q(x, y)] \)
   (4) For sets \( A, B, C \subseteq \mathbb{U} \). If \( A \subseteq B \) and \( B \not\subset C \), then \( A \subset C \).
   (5) Let \( m, n \in \mathbb{Z} \). If \( m, n \) are perfect squares, then \( m + n \) is a perfect square.
   (6) Let \( f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{Z} \) be defined by \( f(a, b) = \lceil a + b \rceil \). Then \( f \) is associative.
   (7) Let \( f, g: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+ \) where for all \( x \in \mathbb{Z}^+ \), \( f(x) = x + 1 \) and \( g(x) = \max\{1, x-1\} \). Then \( f \) is a one-to-one and onto function while \( g \) is onto but not one-to-one.
   (8) For an alphabet \( \Sigma \), let \( A, B, C \in \Sigma^* \). Then \( A(B \cap C) \subseteq AB \cap AC \).
   (9) A relation is called a partial order if it is reflexive, symmetric and transitive.
   (10) If \( a \mid (bx + cy) \), then \( a \mid b \) or \( a \mid c \), \( \forall a, b, c \in \mathbb{Z} \).

2. Simplify the following network. (10%)

3. Five speakers (A, B, C, D, and E) are scheduled to present papers in a conference.
   (a) How many ways can this be arranged? (5%)
   (b) How many ways can this be arranged without B speaking before A? (5%)
   (b) How many ways can this be arranged if A speaks immediately before B? (5%)

4. Consider the following directed graph.
   (a) Find the corresponding adjacency matrix. (5%)
   (b) Find the transitive closure of the relation represented by this graph. (5%)
   (c) Identity what properties the relation corresponding to this graph satisfy. Reflexivity?
Irreflexivity? Symmetry? Antisymmetry? Asymmetry? (8%)

(d) Does this graph contain an Euler cycle? If yes, show the cycle. If not, explain why? (7%)

5. Let input and output alphabet \( I = O = \{a, b\} \). Construct a state diagram for a finite state machine that can recognize \((aa*bb*)^*\). (10%)

6. Two integers \( a \) and \( b \) are congruent modulo \( n \) if \( n \mid (a - b) \), denoted as \( a \equiv b \pmod{n} \). Show that congruence modulo \( n \) is an equivalence relation on \( \mathbb{Z} \). (10%)
1. (10%) (a) Write a truth table for the Boolean function \( F(X,Y,Z) = XY + YZ \). (b) Simplify the function so that it contains the minimal number of terms.

2. (10%) Consider a two-dimensional array \( M[0][0][0][0] \). If element \( M[0][0][0][0] \) is stored at memory address 2000 (in hexadecimal format), what is the address (in hexadecimal format) of element \( M[4][5] \) if \( M \) is a (a) row major or (b) column major array?

3. (10%) Consider the following simple 8-bit floating point notation. A 0 in the sign bit means that the value stored is nonnegative. Values stored in the exponent and mantissa fields are using 2 complement method. (a) What is the decimal value corresponding to bit pattern 11101011? (b) When we want to store \( (3.625)_{10} \) in this notation, we will encounter a truncation error. What part of the value being stored is lost?

4. (5%) Ada uses \texttt{end if} to mark the end of the \texttt{if} clause. Please complete the following program by inserting two \texttt{end if} so that it matches the flow chart shown in the left-hand side.

\begin{verbatim}
if (sum == 0) then
  if (count == 0) then
    result = 0;
  else
    result = 1;
else
  result := 0
end if
end if
\end{verbatim}

5. (10%) Write a C function \texttt{void swap(int **a, int **b)} that exchanges the contents of two integer pointers \( a \) and \( b \).

6. (10%) Show the step-by-step construction of an AVL tree resulting from the insertion of the following sequence of keys: \( 3, 5, 6, 4, 1, 2, 8, 7, 9 \).
7. (15%)

(1). Compare three heaps: (a) max heap (b) deap (c) min-max heap.

(2). Show the final heap trees of (a) max heap (b) deap (c) min-max heap after inserting nine integers 3, 5, 6, 4, 1, 2, 8, 7, 9.

8. (10%) Given the following forest, (1) transform the forest into a binary tree; (2) find the postorder and inorder traversals of the forest.

9. (15%) Given \( n \) distinct elements \( n \gg 15 \), design a comparison-based algorithm which requires at most \( 7n \) comparisons to find the 15\(^{th}\) largest element.

(1) Briefly show the comparison-based algorithm.

(2) According to the above algorithm, write a program in C to find the 15\(^{th}\) largest element.

10. (5%) Use the following program to find the traversal of the following tree. (only show the output)

```c
typedef struct node {
    int data;
    tree_pointer left_child, right_child;
} ;

type struct node *tree_pointer;

void order (tree_pointer ptr)
{
    If (ptr) {
        If (ptr-> data % 2 ==0) {
            order(ptr-> left_child); order(ptr->right_child);
        } else {
            order(ptr-> right_child); order(ptr-> left_child);
        }
        printf(\d? ptr->data);
    }
}
```
1. (12%) `read()` is a system call and `fread()` is a standard C library call. (a) Explain how they work in the runtime environment. (b) Discuss their performance issues and indicate in what case one is better than the other.

2. (12%) Consider the following processes, with the arrival time and CPU burst time given in milliseconds.

<table>
<thead>
<tr>
<th>Process</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival Time</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Burst Time</td>
<td>10</td>
<td>26</td>
<td>4</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

For each of the (a) first-come, first-served (FCFS), (b) preemptive shortest job first (SJF), and (c) round robin (RR, quantum = 10 milliseconds) scheduling algorithms for this set of processes, draw the Gantt chart and determine the average waiting time and the average turnaround time.

3. (12%) (a) Explain how to implement mutual-exclusion with hardware-supported `Swap()` instruction. (b) Explain how to implement distributed mutual-exclusion with a ring-based algorithm.

4. (12%) How does the two-phase locking protocol manage locks in the growing phase when an operation accesses an object within a transaction?

5. (12%) In peer-to-peer (P2P) applications a node may act as both a client and a server. There are three common models to implement P2P software: (a) a single-threaded process with non-blocking I/O (b) multiple single-threaded processes with inter-process communication, and (c) a multi-threaded process with shared memory. Discuss strengths and weaknesses of each model.

6. (12%) Flash memory is non-volatile computer memory that is primarily used in memory cards and flash drives for general storage. As its capacity grows, flash drives are commonly equipped as replacement of traditional hard disk drives. (a) In the design of file systems, what are the major features that make flash drives different from hard disk drives? (b) What are the issues if we format a flash drive as a FAT file system?

7. (12%) Consider a number of requests with associated cylinders and deadlines (in milliseconds)

<table>
<thead>
<tr>
<th>Request</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>R5</th>
<th>R6</th>
<th>R7</th>
<th>R8</th>
<th>R9</th>
<th>R10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder</td>
<td>77</td>
<td>95</td>
<td>25</td>
<td>28</td>
<td>100</td>
<td>90</td>
<td>50</td>
<td>77</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>Deadline</td>
<td>57</td>
<td>300</td>
<td>250</td>
<td>88</td>
<td>85</td>
<td>110</td>
<td>299</td>
<td>300</td>
<td>120</td>
<td>212</td>
</tr>
</tbody>
</table>

The disk head moving within 100 cylinders, numbered 1 to 100, is currently at cylinder 94 and toward cylinder 95. What is the order and the total distance (in cylinders) that the disk arm moves to satisfy all the requests for (a) SCAN disk scheduling, and (b) SCAN-EDF disk scheduling? Assume requests with deadlines occurring within 100 milliseconds of each other will be batched.

8. (16%) Consider the following page reference string

0, 1, 2, 3, 1, 4, 1, 5, 3, 4, 1, 4, 3, 2, 3, 1, 2, 0, 1, 2

for a memory with three frames initially empty. How many page faults would occur for the following page replacement algorithms? (a) LRU (b) LFU (c) FIFO (d) optimal