一、 選擇題 (78%) (每題六分):

※（請從試卷最後的答案欄中挑選正確答案編號，依題號順序於答案卷作答）

1. Find the equation of the tangent line to \( f(x) = \frac{1}{x} \) at \( x = 2 \).

2. Find a value of \( c \) for which the conclusion of the Mean Value Theorem is true. 
   \( f(x) = x^3 - 2x \) on the interval \([0, 1]\).

3. Find the limit \( \lim_{x \to \infty} x^{3/\sqrt{x}} \).

4. Find the speed of the particle when it crosses the \( x \)-axis the first time. The particle's position is given by the parametric equations:
   \[
   \begin{align*}
   x(t) &= \sin t(2 - \cos t) \\
   y(t) &= 2\sin t - 1 
   \end{align*}
   \], \( t \geq 0 \).

5. Use the Fundamental Theorem to compute the integral exactly. 
   \[ \int_{0}^{\sqrt{2}} \frac{4}{\sqrt{1-x^2}} \, dx \]

6. Evaluate the integral. 
   \[ \int e^{-t} \sin(x) \, dx \]

7. Evaluate the integral. 
   \[ \int \frac{x-1}{x^2-7x+10} \, dx \]

8. Find the area of the region bounded by the curves. \( x = 4 - y^2, \ x - y = -2 \)

9. Find the volume of the solid with cross-sectional area \( A(x) = \pi(8-x)^2, \ 0 \leq x \leq 8 \).

10. Solve the initial value problem. 
    \( y' = \frac{x}{\cos y}, \ y(0) = 0 \)

11. Find \( \frac{\partial f}{\partial x} \) if \( f(x, y) = 3x^3 - xy^5 + x^2y + 5y^3 \).

12. Compute the directional derivative of \( f(x, y) = x^3 + y^2 - xy + 6 \) at the point \((2, 1)\) in the direction of the unit vector \( \mathbf{u} = \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \).

13. Find the volume of the solid formed by revolving the region bounded above by the parabola \( y = 1 - x^2 \) and below by the \( x \)-axis, about the \( x \)-axis.
二、計算題：

1. (10%) Find a power series representation of \( f(x) \) about \( c = 0 \) and give the interval of convergence. \( f(x) = \frac{7}{1+x} \)

2. (12%) Find the Fourier series of the function on the given interval.

\[
f(x) = \begin{cases} 
\frac{1}{2}, & -1 < x < 0 \\
1, & 0 < x < 1
\end{cases}
\]

※ <<選擇題答案欄>>

<table>
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<tr>
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<tbody>
<tr>
<td>1.</td>
<td>( \frac{1}{3} \log</td>
<td>x-5</td>
<td>- \frac{4}{3} \log</td>
<td>x-1</td>
<td>+ c )</td>
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<td>4.</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>5.</td>
<td>( \frac{3}{\sqrt{2}} )</td>
<td>6.</td>
<td></td>
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<td>7.</td>
<td>( \frac{1}{\sqrt{3}} )</td>
<td>8.</td>
<td>( \frac{125}{6} )</td>
<td>9.</td>
<td>( \frac{2\sqrt{3}}{2\sqrt{3}-1} )</td>
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<td>10.</td>
<td>does not exist</td>
<td>14.</td>
<td>( 2x - 2y )</td>
<td>15.</td>
<td>( y = -\frac{1}{4}x + 1 )</td>
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<tr>
<td>19.</td>
<td>( y = \tan^{-1} \left( \frac{\pi x^2}{2} \right) )</td>
<td>20.</td>
<td>( \frac{512\pi}{3} )</td>
<td>21.</td>
<td>( \frac{\sqrt{6} - \sqrt{2}}{2} )</td>
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<td>25.</td>
<td>( \frac{e^{-x}}{2} (\sin x + \cos x) + c )</td>
<td>26.</td>
<td>( 2e^{-x}(\sin x - 2\cos x) + c )</td>
<td>27.</td>
<td>( \frac{3\pi}{2} )</td>
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背後尚有試題 第 2 頁 共 2 頁
1. The mass in the figure slides on a frictionless surface. If \( m = 2 \, \text{kg} \), \( k_1 = 800 \, \text{N/m} \) and \( k_2 = 500 \, \text{N/m} \), the frequency of oscillation (in Hz) is approximately (6 points)

   a. 6
   b. 2
   c. 4
   d. 8
   e. 10

2. In an adiabatic free expansion (6 points)
   a. no heat is transferred between a system and its surroundings.
   b. the pressure remains constant.
   c. the temperature remains constant.
   d. the volume remains constant.
   e. the process is reversible.

3. A negatively charged particle is moving in the \(+x\)-direction when it enters a region with a uniform electric field pointing in the \(+x\)-direction. Which graph gives its position as a function of time correctly? (Its initial position is \( x = 0 \) at \( t = 0 \).) (6 points)

   a. 
   b. 
   c. 
   d. 
   e.
4. The sky is blue because of
   a. interference.
   b. diffraction.
   c. scattering.
   d. dispersion.
   e. polarization.

5. A waiter in a restaurant fills a pitcher full of water and ice so that water would spill out if any
   more were added. As the ice starts to melt
   a. the water level in the pitcher falls.
   b. the water level in the pitcher remains the same.
   c. water starts to flow out the spout of the pitcher.
   d. the pressure on the bottom of the pitcher decreases.
   e. the pressure on the bottom of the pitcher increases.

6. A student holds a tuning fork oscillating at 256 Hz. He walks toward a wall at a constant speed
   of 1.50 m/s. Assume the speed of sound in air is 343 m/sec. (a) What beat frequency does he
   observe between the tuning fork and its echo? (b) How fast must he walk away from the wall
   to observe a beat frequency of 5.00 Hz?

7. A projectile of mass \( m \) moves to the right with a speed \( v_i \). The projectile strikes and sticks to
   the end of a stationary rod of mass \( M \), length \( d \), pivoted about a frictionless axle through its
   center. (a) Find the angular speed of the system right after the collision. (b) Determine the
   fractional loss in mechanical energy due to the collision. (\( I_{CM} = \frac{1}{12}ML^2 \))

8. A crate of weight \( F_g \) is pushed by a force \( P \) on a horizontal floor. (a) If the coefficient of static
   friction is \( \mu_s \) and \( P \) is directed at angle \( \theta \) below the horizontal, what is the minimum value
   of \( P \) that will move the crate? (b) Find the minimum value of \( P \) that can produce motion when
   \( \mu_s = 0.400, F_g = 100 \) N, and \( \theta = 30.0^\circ \).

9. A laser beam in air is incident at an angle of 45.0° from the vertical onto a solution of corn
   syrup in water. If the beam is refracted to 30.0° from the vertical, (a) what is the index of
   refraction of the syrup solution? Suppose the light is red, with vacuum wavelength 632.8 nm.
   Find its (b) frequency, (c) speed, and (d) wavelength in the solution.
10. A conducting rod of length $\ell$ moves on two horizontal, frictionless rails, as shown in the figure. If a constant force of 1.00 N moves the bar at 3.00 m/s through a magnetic field $\mathbf{B}$ that is directed into the page, (a) what is the current through the 12.0-$\Omega$ resistor $R$? (b) What is the rate at which energy is delivered to the resistor? (c) What is the mechanical power delivered by the force $\mathbf{F}_{app}$? (18 points)