1. (10%) Determine the frequency counts for all statements in the following program segments.

```c
1. i=1;
2. while (i<=n) {
3.   x++;
4.   i++; }
```

2. (10%) In the following, we list a matrix multiplication function which computes the matrix 

\[ C = A \times B. \]

Please show the statements (a) and (b).

```c
void mult (int A[][max_size], int B[][max_size], int C[][max_size])
{
    int i, j, k;
    for (i=0; i<max_size; i++)
        for (j=0; j< max_size; j++) {
            C[i][j]=0;
            for (k=0; ______________)
                ___________________
            }
            for (k=0; ______________)
                ___________________
        }
}
```

(a) ______________
(b) ______________

3. (10%) Given a string \( s = \text{baabaabb} \) and a pattern \( \text{pat} = \text{abb} \)? let try to match pattern \( \text{pat} \) in the string \( s \) by using Knuth-Morris-Pratt Algorithm.

Definition: If \( p = p_0p_1\ldots p_{n-1} \) is a pattern, then its failure function, \( f \), is defined as

\[
f(j) = \begin{cases} 
\text{largest } k < j & \text{such that } p_0p_1\ldots p_k = p_{-k-1}p_{-k}\ldots p_{-j} \text{ if such a } k \ge 0 \text{ exists} \\
-1 & \text{otherwise.}
\end{cases}
\]

4. (10%) Given a sparse \( 3 \times 3 \) matrix \( A = \begin{bmatrix} 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 \end{bmatrix} \), please use a link list to represent the matrix \( A \).

5. (10%)

Given the tree in the right figure,

find its (a) postorder traversal  (b) level order traversal
6. (10%) Find the binary tree which has
   inorder sequence: 513968427
   preorder sequence: 615934872

7. (10%) Use the following program to find the traversal of the following tree. (only show the output)

   ```c
   typedef struct node *tree_pointer;
   typedef struct node {
     int data;
     tree_pointer left_child, right_child;
   };
   
   void order (tree_pointer ptr)
   {
     If (ptr) {
       If (ptr-> data % 2 ==0) {
          order(ptr-> right_child); order(ptr-> left_child);
       }
       else { order(ptr-> left_child); order(ptr-> right_child);
       printf(? d?  ptr->data);
     }
   }
   ```

8. (10%) Read a serial of numbers of 9, 6, 5, 2, 10, 7, 8, 3, 4 and 1 in sequence to find:
   (a) max heap tree
   (b) binary search tree

9. (20%)
   (a) Please write a quick-sort program (or algorithm) to sort the array A[1:n].
   (b) Show and explain the time complexities of the quick-sort program in the best case, the average case, and the worst case.
   (c) Apply the quick-sort program to sort A[1:8]=(26, 5, 37, 1, 61,11, 59, 15) in nondecreasing order. Show the action step by step and identify the number of required comparisons.
1. (a) (3%) What is a Hamilton cycle?
(b) (4%) Is there a Hamilton cycle in the following graph? If your answer is yes, show it; otherwise, prove there is no Hamilton cycle.
(c) (3%) What is an Euler trail?
(d) (4%) Is there an Euler trail in the following graph? If your answer is yes, show it; otherwise, prove there is no Euler trail.
(e) (3%) Is the following graph planar? If your answer is yes, show it; otherwise, prove it is nonplanar.
(f) (3%) What is the chromatic number of the following graph? Explain your answer.
(g) (5%) If five different colors are used to color the vertices such that adjacent vertices have different colors, how many different ways can we color the vertices?

2. (a) (5%) Let \( R \) be a partial order on a finite set \( S \). Prove that \( S \) has a minimal element with respect to \( R \).
(b) (5%) Let \( R \) be a total order on a finite set \( S \). Prove that \( S \) has a minimal element with respect to \( R \), and the minimal element is unique.

3. (6%) Let \( n \) be an odd positive integer. Prove that 12 divides \( n^3 + 1 \ln \).

4. There are 5 married couples (husband and wife) to be seated in a row as shown in the following graph.

   1  2  3  4  5  6  7  8  9  10

(a) (4%) In how many ways can they be seated?
(b) (4%) If no man be seated beside another man, in how many ways can they be seated?
(c) (5%) If each couple must be seated together, in how many ways can they be seated?
(d) (5%) If no couple is seated together, in how many ways can they be seated?
(e) (5%) If exactly two men are seated beside their wives, in how many ways can they be seated?

5. (8%) If the 26 letters, A, B, Z are written in a circular array, prove that there must be 5 consecutive consonants.

6. There are ten different presents.
   (a) (4%) If these presents are assigned to 4 people such that each person has at least one present, in how many ways can these 10 presents be assigned?
   (b) (4%) If these presents are packed into 4 same boxes such that each box contains at least one present, in how many ways can these 10 presents be packed?

7. (a) (5%) Write a recurrence relation and initial conditions for the number of ways to group \(2n\) people into pairs.
   (b) (5%) Find a formula for (a).

8. (5%) Let \(a_r\) be the number of ways of spending \(r\) dollars buying balls, if 5 identical black balls (1 dollar each) and 4 identical red balls (3 dollars each) are available. Find the generating function for the sequence \(\{a_r\}\).

9. (5%) Determine if the following statement is tautology.
   \[(p \to q) \to [(p \lor q) \to q]\]