1. Write a recursive function in C that accepts integer \( n \geq 0 \) as an input parameter and returns the value of \( a_n \), where \( a_n \) is a recurrence relation defined below: (15%)

\[
\begin{align*}
a_n &= a_{n-1} + 2a_{n-2}, & n \geq 2 \\
a_0 &= 2 \\
a_1 &= 7
\end{align*}
\]

2. Given a 16-bit unsigned integer \( x \), please state how to clear its last four bits to all 0’s (leaving the rest bits unchanged) by bit masking. You should specify both the bitwise operator in use (AND, OR, or XOR) and the operand be applied. (10%)

3. Suppose that we have arranged some letters into an ordered binary tree as shown below. Where shall we place a new letter if the new letter is (a) F and (b) O? Please indicate your answer as “the left/right child of node X” (10%)

4. Consider inserting the keys 16, 30, 80, 39, 49, 17, 24, 40 into a hash table of length 11 using hash function \( h(k) = k \mod 11 \). (a) Illustrate the result of the insertion using linear probing. (b) What is the number of collisions occurred during the insertion? (10%)

5. (a) Draw a threaded tree for the binary tree shown below. Use broken lines (dashed lines) to represent threads. (b) How many threads are there? (10%)

6. Draw a binary tree such that the inorder traversal of this tree is \( w, v, x, u, h, e, r, f, g \) while the preorder traversal is \( r, u, v, w, x, e, h, f, g \). (10%)
7. A max heap is a complete binary tree such that the key value in each node is no smaller than the key values in its children (if any). (a) If we insert an element with key value 21 into the max heap shown below, what will be the result? (b) If we delete an element from the max heap after the insertion, what will be the result? (10%)

8. What is the result of performing a one-bit left circular shift on a byte with value 5C (in hexadecimal notation)? Give you answer in hexadecimal form. (10%)

9. Write a program using instructions in the instruction set shown below to output the result of $A \times B$, where $A$ and $B$ are two registers storing integer values (assume no overflow). Your program may use an auxiliary register $C$ in the computation. You may also label an instruction as a target of instruction JZ or JNZ. (15%)

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Meaning (X and Y are any two registers; L denotes a label)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADD X, Y</td>
<td>Store the result of $X + Y$ to $X$</td>
</tr>
<tr>
<td>SUB X, Y</td>
<td>Store the result of $X - Y$ to $X$</td>
</tr>
<tr>
<td>INC X</td>
<td>Increase the value of $X$ by one</td>
</tr>
<tr>
<td>DEC X</td>
<td>Decrease the value of $X$ by one</td>
</tr>
<tr>
<td>JZ X, L</td>
<td>Jump to Label L if $X = 0$</td>
</tr>
<tr>
<td>JNZ X, L</td>
<td>Jump to Label L if $X \neq 0$</td>
</tr>
<tr>
<td>OUT X</td>
<td>Output the value of $X$</td>
</tr>
</tbody>
</table>
1. (8%) Let \( x, \ y \in Z \). Find all the integer solutions of
\[ 131x + 32y = 2 \]
for \( 0 < x < 100 \).

2. (8%) How many integer solutions are there to the equation
\[ w + x + y + z = 15 \]
for \( 0 \leq w \leq 4, \ 0 \leq x \leq 5, \ 0 \leq y \leq 6, \) and \( 0 \leq z \leq 7 \)?

3. Let \( w, \ x, \ y, \ z \in Z, \ w, \ x, \ y, \ z \geq 0 \), and \( w \) is an odd integer.
   (a) (4%) Describe how to apply generating-function methods to find the number of solutions of
   \[ w + 2x + 2y + 5z = n. \]
   (b) (4%) If \( n = 30 \), what is the number of solutions in (a)?

4. (8%) Suppose we have an \((n+2) \times (n+1)\) grid of pints like the following one.

```
(0,0)
```

We are interested in paths from \((0, 0)\) to \((n,n+1)\) that move only to the right or the up. Try a combinatorial proof to show that
\[
\binom{2n+1}{n} = \sum_{t=0}^{n} \binom{n}{n+1-t} \binom{n+1}{t}, \text{ for } n \geq 0, \ n \in Z.
\]

5. (8%) Let \( x_1, \ldots, x_{10} \in \mathbb{R} \) and \( x_1, \ldots, x_{10} \in (0, 1) \). Show that the following statement is false:
   For any two numbers, \( x_i \) and \( x_j \), \( i \neq j \), \( |x_i - x_j| \geq \frac{1}{9} \).

6. (8%) Let \( f : A \to B \), with \( A_1, \ A_2 \subseteq A \). Show that \( f \) is one-to-one if and only if for all \( A_1, A_2 \subseteq A, \ f(A_1 \cap A_2) = f(A_1) \cap f(A_2) \).

7. (8%) There are 20 students in a class, all born on different days of January, 1980. Show that there are two students born on the \( i \)th day and the \( j \)th day of January with \( |i - j| = 8 \).
8. John, Mary, Alice, Tom, Jane, and Peter take the school buses to school based on the following conditions.

   (1) John does not get on the same bus with Mary, Alice, Tome, or Jane.
   (2) Peter does not get on the same bus with Mary, Alice, Tom, or Jane.
   (3) Alice does not get on the same bus with Mary or Tom.

(a) (4%) Model this problem as a graph-coloring problem.
(b) (4%) Determine the minimum number of buses to take all these 6 students to school.
(c) (4%) If there are 6 different buses, numbered 1, 2, ..., and 6, how many different arrangements can be made such that all the conditions, (1), (2), and (3), are met?

9. (a) (4%) What is an Euler trail?
(b) (4%) Show that no Euler trail exists in the following graph.

![Graph Image]

10. (a) (4%) What is a bipartite graph?
(b) (4%) Show that if a graph must be colored using at least 3 colors such that adjacent vertices are of different colors, then it is not a bipartite graph.

11. (8%) Show that \( 2^n \geq n^2 - 1 \), for all \( n \geq 3, \ n \in \mathbb{Z} \).

12. (8%) Let \( x, y \in \mathbb{R} \), and \( x \mathrel{R} y \) if and only if \( x = 2y \). Determine the relation \( R \) is reflexive, symmetric, antisymmetric, or transitive. Justify your answer.
1. Please explain the following terms.
   1) Semaphore (5%)
   2) Virtual machine (5%)
   3) RAM disk (5%)

2. Answer the following questions related to threads:
   1) Why does a thread’s control block contain no information about the memory limit? (5%)
   2) Why is the context switch performed for thread more efficient than process? (5%)

3. Consider a computer system with 1MB user space memory and using buddy system as the
   scheme for memory allocation. Initially, all processes are in disk job pool. Give the following
   information:

<table>
<thead>
<tr>
<th>Process</th>
<th>CPU cycle (sec)</th>
<th>Arrival time (sec)</th>
<th>Process size</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>8</td>
<td>0</td>
<td>100K</td>
</tr>
<tr>
<td>P2</td>
<td>4</td>
<td>2</td>
<td>240K</td>
</tr>
<tr>
<td>P3</td>
<td>2</td>
<td>8</td>
<td>64K</td>
</tr>
<tr>
<td>P4</td>
<td>5</td>
<td>6</td>
<td>256K</td>
</tr>
<tr>
<td>P5</td>
<td>10</td>
<td>4</td>
<td>75K</td>
</tr>
</tbody>
</table>

   Consider two CPU scheduling algorithms: FCFS, preemptive SJF, RR with time quantum = 7.
   Please answer the following questions.
   1) Draw the Gantt chart for each algorithm. (12%)
   2) Compute the average waiting time for each algorithm. (6%)
   3) For each scheduling algorithm, show the memory configuration snapshot (i.e., which range
      of memory is used to allocate which process, and which range denote available space) at the
      time when process P4 starts execution. (12%)

4. Consider a system consisting of four resources of the same type being shared by three processes.
   Resources can be requested and released by processes only one at a time. Assume that each
   process needs a maximum of two resources. Show that a deadlock cannot occur. (10%)

5. For the three disk allocation methods, contiguous allocation, linked allocation, and indexed
   allocation, discuss their characteristics from the following aspects. (10%)
   1) fragmentation: suffering internal or external
2) access method: supporting sequential or direct

6. Consider the two dimensional array A:
   
   ```
   int A[][] = new int [50] [50];
   ```

   where A[0][0] is at location 100, A[0][1] at 102, and A[0][2] at 104, etc., in a paged system with pages of size 100 bytes. A small process is in page 0 (locations 0 to 99) for manipulating the matrix; thus, every instruction fetch will be from page 0.

   Assume that there are five pages frames; page frame 1 has the process in it and the other four are initially empty. Consider two different page replacement algorithms: FIFO and LRU, and the following loop:
   ```
   for (int j = 0; j < 50; j++)
       for (int i = 0; i < 50; i++)
           A[i][i] = 0;
   ```

   1) List the page reference string for the execution of the loop. (5%)

   2) For each replacement algorithm, how many page faults are generated by the loop? (8%)

7. Consider the following map of disk usage. Shaded blocks denote allocated space. Please write the results for each of the following free disk space management methods.

   1) Bit vector (4%)
   2) Linked list (4%)
   3) Grouping (4%)

![Disk Usage Map]