Leading Merger in a Stackelberg Oligopoly: Profitability and Consumer Welfare

Chih-Chen Liu
and
Leonard F.S. Wang*
Department of Applied Economics, National University of Kaohsiung, Taiwan

Abstract
This paper examines the effects of obtaining a strategic advantage of becoming the leader in the market on insiders’ incentives to merge and consumer welfare. We show that being the market leader is privately profitable for the merging insiders. We also show that the leading merger would benefit consumers if and only if the number of insiders is sufficiently small.

Keywords: Horizontal merger; Stackelberg competition; Consumer welfare

JEL Classification: D43; L11; L13; L40; L41

*Correspondence: Leonard F. S. Wang, NUK Chair Professor, Department of Applied Economics, National University of Kaohsiung, No. 700, Kaohsiung University Road, Nan-Tzu District 811, Kaohsiung, Taiwan, R.O.C. Tel:886-7-5919322, Fax:886-7-5919320, E-mail address: lfswang@nuk.edu.tw
1. Introduction

The literature on horizontal mergers has been mainly focused on the “merger paradox”. In a symmetric linear Cournot oligopoly with homogenous goods, Salant et al., (1983) showed that mergers which do not involve a vast majority of industry participants are generally harmful for the merging insiders, while benefit the outsiders. Since then a growing literature is devoted to identify alternative models for privately beneficial mergers. For example, in an industry where the number of participants is fixed, Perry and Porter (1985) emphasized on the convexity of cost function the firms can profitably merge and Farrell and Shapiro (1990) focused on synergic effects of mergers.

Horizontal mergers may influence social welfare from various aspects in different market structures. In a Cournot market with no entry, if the merger uses identical production technology with the outsiders, then consumer surplus and social welfare would decrease after merger for a lower industry output. In the presence of free entry and exit of the market, Davidson and Mukherjee (2007) demonstrated that, while it has no impact on industry output and thus consumer surplus, a horizontal merger is both privately and socially beneficial for any degree of cost synergies since the society can produce the same amount of goods in a lower social cost.

In a Stackelberg market, Huck et al., (2001) showed that only mergers between a
leader and a follower are unambiguously profitable. Escrihuela-Villar and Faulí-Oller (2008) extended the analysis allowing cost asymmetries and showed that mergers among followers are privately profitable but socially harmful when the followers are inefficient enough. Heywood and McGinty (2008) examined a merger between a leader and followers and showed that, with convex costs, a profitable merger could harm excluded rivals but benefit the society.

Previous studies examine the effects of horizontal mergers in exogenous competition modes. In this paper, we point towards a new factor, i.e., the effects of obtaining a strategic advantage of becoming the leader in the market on the insiders’ incentives to merge. It would be plausible that if a horizontal merger leads to economies of scale in production and/or economies of scope in synergies among insiders as suggested by Farrell and Shapiro (1990), the merger may obtain a dominant position and may enable the merger become the leader in the market. However, in order to focus on the merger’s strategic effect of becoming the market leader on the equilibrium outcome, we assume that, except for the sequence of moves, the merger is identical to other firms in the market. We show that being the leader is privately profitable for the merging insiders. We also show that, even though there is no innovation, cost synergy, and free entry, the leading merger could benefit consumers if and only if the number of insiders is sufficiently small.
2. Profitability of a Leading Merger

Consider an industry with $m + k$ symmetric firms producing homogeneous goods at a constant marginal cost $c > 0$. Firm $i = 1, \ldots, m$ is an insider that may agree to merge and firm $i = m + 1, \ldots, m + k$ is an outsider. The inverse market demand is given by $P(Q)$, where $P$ is price, $Q$ is industry output, and $P'(Q) < 0$.

Conditional on the insiders merge or not, these firms compete in different modes. If all firms behave non-cooperatively, i.e., there is no merger, we assume that these firms compete like Cournot oligopolists. In the Cournot equilibrium, each firm $i$ chooses its output $q_i$ to maximize its profits, given its rivals’ outputs. Firm $i$’s first order condition, $\partial \pi_i / \partial q_i = 0$, is

$$P(Q) - c + q_i P'(Q) = 0, \quad i = 1, \ldots, n.$$  

(1)

A Cournot equilibrium is a vector of $(q_1^*, \ldots, q_{m+k}^*)$ such that equation (1) holds for all $m + k$ firms. Due to the symmetry among these firms, we focus on a symmetric equilibrium in which the equilibrium output of the $i$th firm, $i = 1, \ldots, m + k$, and the industry output are represented as $q_i^* = q^*$ and $Q^* = (m + k)q^*$, respectively. The $i$th firm’s equilibrium profit is $\pi_i = [P(Q^*) - c]q^*$, $i = 1, \ldots, m + k$.

If the $m$ insiders merge, we assume that the merger results in two changes. First, the insiders act like a single firm and choose their aggregate output to maximize their joint profits. Second, the merger obtains a strategic advantage of becoming the leader.
in the market.

We solve a subgame perfect Nash equilibrium (SPNE) of the game with the following sequence of moves. In the first stage, the insiders merge and choose their aggregate output $q_m$ to maximize their joint profits. In the second stage, each outsider simultaneously chooses its output $q_i$, $i = 1, ..., k$, to maximize its own profit. The game is solved by backward induction.

In the second stage of the game, an outsider $i$’s first order condition, 

\[
\frac{\partial \pi_i}{\partial q_i} = 0, \quad i = 1, ..., k.
\]

Consider the effect of a change in the merger’s output $q_m$ on outsider $i$’s output. We assume that each firm’s reaction curve slopes downward, i.e., $P'(Q) + q_iP''(Q) < 0$ for the merger and all outsiders $i = 1, ..., k$. This assumption would hold if the industry demand satisfies $P'(Q) +QP''(Q) < 0$.\(^1\) For the symmetry among the outsiders, from equation (2), we have

\[
\frac{dq_i}{dq_m} \equiv R_i = -\frac{p'(Q) + qIp''(Q)}{p'(Q) + k[p'(Q) + qIp''(Q)]}, \quad i = 1, ..., k.
\]

It is clear, from equation (3), that $-1 < R_i < 0$, $i = 1, ..., k$. That is, if the merger increases its output, each outsider would decrease its output, but less than the merger’s expansion. Denote $E \equiv QP''(Q)/P'(Q)$ for the elasticity of the slope of the

\(^1\) This assumption is a standard and weak assumption in analysis, see Dixit (1986), Shapiro (1989), and Farrell and Shapiro (1990).
inverse demand and $s_i \equiv q_i/Q$ for outsider $i$’s market share. Then, we have

$$R_i = -\frac{1 + s_i E}{1 + k[1 + s_i E]}, \quad i = 1, \ldots, k. \quad (4)$$

With linear demand and constant marginal cost, $E = 0$ and $R_i = -1/(1 + k)$.

Turning to the first stage, the merger’s first order condition, $\partial \pi_m / \partial q_m = 0$, is

$$P(Q) - c + q_m P'(Q)\left(1 + \sum_{i=1}^k R_i\right) = 0. \quad (5)$$

A Stackelberg equilibrium is a vector $(q_m^*, q_1^*, \ldots, q_k^*)$ such that equation (2) holds for all outsiders and equation (5) holds for the merger. For the symmetry among the outsiders, we will focus on the symmetric equilibrium among the outsiders in which the equilibrium output of the merger, the equilibrium output of each outsider, and the industry output are $q_m^*$, $q_i^*$, and $Q^*$.

Our first proposition indicates that obtaining a strategic advantage of being the leader in a market provides incentives for the insiders to merge.

**Proposition 1.** **Being the leader in the market is profitable for the merging insiders.**

**Proof.**

See the Appendix. □

Apart from the merger paradox suggested by Salant et al., (1983), Proposition 1 suggests that, for any number of insiders, a merger is privately profitable if it obtains
a strategic advantage of becoming the leader in the market.

3. **Consumer Welfare Effect of a Leading Merger**

Consider the objective of antitrust policy is to maximize consumer welfare. While a merger generally changes all firms’ outputs, consumers care about the changes in industry output only. Thus, for policy purposes, consumer welfare effect of a leading merger has to do with the change in equilibrium industry output after merger.

In order to examine the effect of the leading merger on equilibrium industry output, in the following Lemma, we first show the relationship between the output of the merger and the industry output.

**Lemma.** *Industry output moves in the same direction as the merger’s output, but less.*

**Proof.**

See the Appendix. □

Next, to examine whether the industry output would increase after merger, by our Lemma, we only need to show whether the merging insiders would increase their aggregate output after merger. Proposition 2 gives a necessary and sufficient condition
on the number of insiders for industry output to increase with the leading merger.

**Proposition 2.** The leading merger would increase industry output and benefit consumers if and only if \( m < 1/(1 + \sum_{i=1}^{k} R_i) \).

**Proof.**

See the Appendix. \( \Box \)

Now, we illustrate the results in a market with linear market demand \( P(Q) = 1 - Q \) and a constant marginal cost \( c \). Before merger, the equilibrium output of each firm, industry output, and profit of each firm are \( q^* = (1 - c)/(1 + m + k) \), \( Q^* = n(1 - c)/(1 + m + k) \), and \( \pi^* = (1 - c)^2/(1 + m + k)^2 \). In this case, \( R_i = -1/(1 + k) \). After merger, the equilibrium output of the merger, output of each outsider, industry output, profit of the merger, and profit of each outsider are

\[
q_m^{**} = (1 - c)/2, \quad q_o^{**} = (1 - c)/2(1 + k), \quad Q^{**} = (1 + 2k)(1 - c)/2(1 + k),
\]

\[
\pi_m^{**} = (1 - c)^2/4(1 + k), \quad \pi_o^{**} = (1 - c)^2/4(1 + k)^2.
\]

Comparing the aggregate profits of the insiders before and after merger, we find that the merger is privately profitable, i.e., \( \pi_m^{**} - m\pi^* > 0 \), for all \( m > 0 \).

Furthermore, by comparing industry outputs before and after merger, we find that
industry output and the resulting consumer welfare increase if and only if \( m < 1 + k \).

4. **Conclusion**

We examine the effects of obtaining a strategic advantage of becoming the leader in the market on the insiders’ incentives to merge and consumer welfare. We show that being the leader is profitable for the merging insiders. Thus, we put forward a new factor, viz., obtaining a strategic advantage of being the leader in the market increases the insiders’ incentives for a merger. We also show that the leading merger would benefit consumers if and only if the number of insiders is sufficiently small.
Appendix

Proof of proposition 1

Suppose that the insiders merge and choose their aggregate output \( q_m \) to maximize their joint profits, and then let each outsider’s output \( q_i, \ i = 1, \ldots, k \), adjust to establish a Stackelberg equilibrium. Denote \( q^* \) and \( Q^* \) for the equilibrium output of each firm and industry output before merger, respectively; \( q_{m}^{**} \) and \( Q^{**} \) are equilibrium aggregate output of the insiders and industry output after merger, respectively. Since the insiders choose their aggregate output \( q_{m}^{**} \) to maximize their joint profits, we have \( [P(Q^{**}) - c]q_{m}^{**} \geq [P(Q) - c]q_m \), for all \( q_m \).

If the merger produces the same aggregate output as before merger, i.e., \( q_m = mq^* \), it can be shown that \( P(Q) = P(Q^*) \). Thus, it is clear that

\[
[P(Q^{**}) - c]q_{m}^{**} \geq [P(Q) - c]q_m = [P(Q^*) - c]mq
\]

Proof of Lemma

For any outsider \( i \), from equation (3), \( dq_i = R_i dq_m, \ i = 1, \ldots, k \). Adding up for all \( k \) outsiders, we have \( \sum_{i=1}^{k} dq_i = \sum_{i=1}^{k} R_i dq_m \). Adding \( dq_m \) to this equation gives \( dQ = dq_m + \sum_{i=1}^{k} dq_i = (1 + \sum_{i=1}^{k} R_i) dq_m \). Thus, we have

\[
\frac{dQ}{dq_m} = 1 + \sum_{i=1}^{k} R_i = \frac{p'(Q)}{p''(Q) + k[p'(Q) + q_1p''(Q)]} < 1.
\]
Proof of Proposition 2

If the insiders produce their aggregate output at their pre-merger level $mq^*$ after merger, each outsider would produce $q^*$ and the industry output and market price are $Q^*$ and $P(Q^*)$, respectively. In this case, the merger’s marginal revenue is

$$P'(Q^*) + mq^*P'(Q^*)(1 + \sum_{i=1}^{k} R_i).$$

The merger would choose to increase its output if $P'(Q^*) + mq^*P'(Q^*)(1 + \sum_{i=1}^{k} R_i) > c$. From equation (1), we have $P'(Q^*) + q^*P'(Q^*) = c$. Therefore, signing $mq^*P'(Q^*)(1 + \sum_{i=1}^{k} R_i) - q^*P'(Q^*)$ or $q^*P'(Q^*)[m(1 + \sum_{i=1}^{k} R_i) - 1]$ is required to determine whether the merger would increase its output after merger. It can be shown that $q^*P'(Q^*)[m(1 + \sum_{i=1}^{k} R_i) - 1] > 0$ if and only if $m < 1/(1 + \sum_{i=1}^{k} R_i)$. □
References


