Profit-Raising Entry in Vertically Related Markets

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Abstract
We extend the standard vertical oligopoly model (Salinger, 1988; Ghosh and Morita, 2007) to allow for free entry in the upstream sector, and R&D and knowledge spillover in the downstream sector. An increase in the number of firms lowers industry profit is the common wisdom which may not hold in vertical structure of production. We provide a complementary reasoning, and find that aggregate downstream profit can increase with downstream entry when the knowledge-spillover effect and the entry cost are moderate.

Keywords: Cournot competition; Input markets; Free entry; Industry profits
JEL classifications: D43; L11; L13

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1. Introduction

In the standard Cournot model of oligopoly, as the number of firms competing in the market increases, industry profits (the sum of the firms’ profits) decrease through increased product market competition. The nature of the relationship between the number of firms and industry profits influences the incentives for firms. For instance, when firms collude in a product market, they control their own quantities supplied like a monopolist, because their monopolistic behavior maximizes the industry profits.

Common wisdom suggests that entry reduces profits of incumbent firms. Naylor (2002) explored a situation in which industry profits increase as the number of downstream firms increase. However, the structure of his model is quite different from ours. He considered a quantity-setting model in which input prices are not exogenous but are determined by bargaining in a bilateral oligopoly. A pair made up of a labor union (an upstream firm) and a downstream firm bargains for the wages of the laborers (the wholesale price). Matsushima (2006) showed that when upstream firms compete in quantity and freely enter the input markets, competition among downstream firms reduces the input price (the marginal cost of downstream firms). The industry profits of downstream firms competing in quantity may then increase with the number of downstream firms. Mukherjee and Zhao (2009) demonstrated that if the incumbents differ in marginal costs and the entrants behave like Stackelberg followers, then entry may benefit the cost efficient incumbents while hurting the cost inefficient ones. The total outputs of all incumbents may be higher under entry. In a successive Cournot oligopoly without endogenously considering technology choice, Mukherjee et al., (2009) considered the welfare effects of entry in the final good market with no scale economies but with cost difference between the firms. In particular, if the input market is very concentrated, entry in the final good market increases welfare; while if the
input market is not very concentrated, entry in the final good market may reduce welfare if the entrant is moderately cost inefficient. Hence, entry in the final good market is more desirable if (1) the input market is very concentrated or (2) the cost difference between the incumbents and the entrant is either very small or very large. It follows from their analysis that entry increases the profits of the incumbent final good producers if their marginal costs are sufficiently lower than the entrant’s marginal cost. The above papers did not consider the implication of R&D investment by the downstream firms on industry profit in a successive Cournot oligopoly at free entry.

In Taiwan’s telecommunication industry, for example, cellular phones are provided by the competing downstream firms that are designing more-friendly use technology, with the important chips and components supplied by the competing upstream firms, that then forms a sustainable supply chain. This real-world industry scenario motivated us to write this paper. In a vertical structure, where entry occurs in the upstream sector and the downstream firms engage in R&D investment; thus determining the production technologies depending on the market structure, we provide new insights to the industry profit effect of free entry in the upstream sector. We show that an increase on the number of downstream firms may more likely be industry profit-raising, when the knowledge spillover effect and the entry cost for the upstream firms are moderate. The profit-enhancing effect associated with knowledge spillover dominating the profit-reducing effect is the economic reason for sustaining the non-falling industry profit in the downstream sector.

The remainder of this paper is organized as follows. We present the basic model in Section 2. Section 3 provides the case where the downstream firms commit to the technology choice before input price determination of the upstream firms and see how the industry profits will be affected by the number of the downstream firms in both general demand and linear demand cases. Section 4 concludes the paper.
2. The Basic Model

Consider an economy with successive Cournot oligopoly as in Greenhut and Ohta (1976), Salinger (1988), Matsushima (2006), Ghosh and Morita (2007). We assume that there are a large number of firms that produce homogeneous good by using an intermediate good produced in an imperfectly competitive market with \( m \) upstream firms, where each upstream firm’s marginal cost of production is \( c \). We consider two types of demands. One is general demand expressed by \( P(Q) \) with \( P'<0 \) and \( P''\leq 0 \), where \( P \) is price and \( Q \) is the total output. The other is linear demand expressed by \( P = a - Q \). Each final good producer decides whether to enter the market. If an intermediate good producer enters the market, it needs to incur a fixed entry cost, \( K \). The number of intermediate good producer is determined by the zero profit condition. Hence, entry in the intermediate good market occurs until the net profit of an intermediate good producer is zero.

We assume that each downstream firm requires one unit of the intermediate good to produce one unit of the final good. Each downstream firm purchases the intermediate good from the upstream sector at a price, \( w \), and incurs a cost \( z \) to convert the intermediate good to the final good. However, each final good producer, which enters the market, invests \( x \) amount in R&D to reduce the cost of converting the intermediate good to the final good to \( (z - x) \). We have considered for analytical convenience that investment in R&D reduces the cost of converting the intermediate good to the final good.\(^1\)

Haruna and Goel (2011) employed a three-stage game model with cost-reducing research and development (R&D) that is subject to spillover to consider the problem of excess entry, and demonstrated that knowledge spillover among firms holds the key to whether established results regarding socially inefficient entry hold. Specifically, excessive entry occurs as long as research spillovers are relatively small, but that is not necessarily the

\(^1\) Our qualitative results hold if R&D reduces input coefficient in the final goods producers’ production technology, instead of reducing the cost of converting the intermediate goods to the final goods.
case with large spillovers. We also allow knowledge spillover under R&D competition in this paper. If there are \( n \) final good producers, we assume that knowledge spillover to the \( i \)th final good producer, \( i = 1, 2, \ldots, n \), is \( \gamma \sum_{d=1}^{n} x_d \), where \( \gamma \in [0, 1/2] \) indicates the degree of knowledge spillover. Hence, the total cost reduction of the \( i \)th firm, \( i = 1, 2, \ldots, n \), through R&D is \( (x_i + \gamma \sum_{d \neq i}^{n} x_d)^2 \). We also assume that R&D is costly and the cost of R&D to each final good producer is \( R(x_i) = x_i^2 / 2 \).

We consider the following game. At stage 1, the intermediate good producers decide whether to enter the market. At stage 2, the upstream firms take their output decisions simultaneously, and the price of the intermediate good is determined from the derived demand for the intermediate good. At stage 3, the final good producers, determine the R&D investment. At stage 4, the final good producers, which have entered the market, produce like Cournot oligopolists, and the profits are realized. We solve the game through backward induction.

### 3. R&D Competition and Industry Profits

#### 3.1 General demand case

We first analyze the Cournot competition in stage 4 subgame, where the \( n \) final good producers enter the market, the \( i \)th final good producer, \( i = 1, \ldots, n \), maximizes the following expression to determine its output:

\[
\pi_i = \left( P - w - z + x_i + \gamma \sum_{d \neq i}^{n} x_d \right) q_i - x_i^2 / 2. \tag{1}
\]

The first-order conditions for profit maximization are

\[
\frac{\partial \pi_i}{\partial q_i} = P - w - z + x_i + \gamma \sum_{d \neq i}^{n} x_d + P' q_i = 0. \tag{2}
\]

Following Ghosh and Morita (2007), we make the following assumptions.

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2 This concept of knowledge spillover is similar to D’Aspremont and Jacquemin (1988).

3 This specification is a special case of D’Aspremont and Jacquemin (1988) without considering the efficiency of R&D while taking technology spillover and input market into consideration.
Assumption 1. (i) \( \lim_{Q \to 0} [P(Q) + QP'(Q)] = P_0 \) and (ii) \((1 + n)P'(Q) + QP''(Q) < 0\) for all \( Q > 0 \).

Assumption 1 guarantees that the system of Eq. (2) yields a unique solution. From Eq. (2), the unique and symmetric Nash equilibrium output is

\[
q_i^*(x_i, w) = ... = q_n^*(x_n, w) = \hat{q}_d(x_d, w).
\]

In a symmetric equilibrium, from Eq. (2), we have the following comparative static results\(^4\)

\[
\frac{dq_i^*}{dx_i} > 0, \quad \frac{dq_i^*}{dn} < 0, \text{ and } \frac{dq_i^*}{d\hat{q}_d} < 0.
\]

Next we consider stage 3 subgame, the first-order conditions for profit maximization are

\[
\frac{\partial \pi_i}{\partial x_i} = (P' \frac{\partial q_i^*}{\partial x_i} + 1)q_i^* - x_i = 0.
\]

(3)

From Eq. (3), the unique and symmetric Nash equilibrium R&D investment is

\[
x_i^*(w) = ... = x_n^*(w) = \hat{x}_d(w).
\]

In a symmetric equilibrium, from Eq. (3), \( \frac{dx_i^*}{dn} < 0 \).

For any given \( w \in (0, P_0) \), adding the first-order conditions and rearranging the terms yields

\[
w = P - z + x_i^* + \gamma \sum_{d=1}^{m} x_d^* + P' q_i^*.
\]

Note that \( q_i^* \) and \( x_i^* \) are functions of \( w \). Letting \( q_j \) and \( Q_l \) denote the amount of intermediate product produced by upstream \( j \) and total industry, we have \( Q_l(w) \equiv q_j + \sum_{j \neq u}^{m} q_u = nq_i^* \).

In stage 2 subgame, the profit function of upstream firm is

\[
\pi_j = (w(Q_l(w)) - c)q_j, \quad j = 1, 2, ..., m.
\]

(4)

Each upstream firm chooses its output to maximize its profit, and the first-order conditions are

\[^4\text{See Appendix.}\]
Assumption 2. (i) \( \lim_{Q_l \to 0} [w(Q_l) + Q_l w(Q_l)] = P_0 \) and (ii) \((1 + m)w'(Q_l) + Q_l w''(Q_l) < 0\) for all \(Q_l > 0\).

Note that we have \(w'(Q_l) < 0\) and \(w'(Q_l) \leq 0\) for all \(Q_l > 0\). Following Ghosh and Morita (2007), assumption 2 guarantees the system of Eq. (5) yields a unique solution, \(q^*_j = ... = q^*_m = \hat{q}_u\) which depends on the numbers of \(n\) and \(m\). In a symmetric equilibrium, from Eq. (5), we have

\[
w^* = c - w' q_j , \quad \text{and} \quad \frac{dw^*}{dn} > 0.
\]

In stage 1 subgame, at free entry equilibrium of upstream firms, each upstream firm’s profit is given by

\[
\pi_j = (w(m\hat{q}_u) - c)\hat{q}_u - K = 0.
\] (6)

We then obtain

\[
w^E = c + \frac{K}{\hat{q}_u}.
\] (7)

**Proposition 1:** As the number of downstream firms increases, the input price \(w^E\) decreases at upstream free-entry equilibrium.

**Proof:** We have \(\frac{dw^E}{dn} = -\kappa \frac{d\hat{q}_u}{\hat{q}_u^2} < 0\).

We infer that as the number of downstream firm increases, the input price \(w^E\) decreases at free entry equilibrium, which is called the input price-reducing effect at upstream free-entry equilibrium. The profit of downstream firm at upstream free-entry equilibrium is

\[
\pi_i^* = (P - w^E - z + x_i^* + \gamma(n - 1)x_i^*)q_i^* - x_i^{*2}/2
\]

\[
= (P - w^E - z + \gamma(n - 1) + 1)x_i^*)q_i^* - x_i^{*2}/2
\]

Next, we want to see how the entry of downstream firms influences the profit of downstream firm. Taking differentiating to \(\pi_i^*\) with respect to \(n\), we have
\[
\frac{d\pi_i^*}{dn} = \pi_i^* + n\left(P - w^E - z + (\gamma(n - 1) + 1)x_i^*\right) \frac{dq_i}{dn}
\]

\[
\begin{align*}
&(+)
\quad
\begin{cases}
(+) \\
(-)
\end{cases}
\end{align*}
\]

\[
+ q_i^* \left( \frac{dp}{dn} - \frac{dw^E}{dn} + (\gamma(n - 1) + 1) \frac{dx_i^*}{dn} + \gamma x_i^* \right) - x_i^* \frac{dx_i^*}{dn}.
\]

\[
\begin{align*}
&(+)
\quad
\begin{cases}
(?) \\
(+)(-)\end{cases}
\end{align*}
\]

From Eq. (8), we find that

\[
\frac{d\pi_i^*}{dn} > 0, \text{ if } \gamma > \hat{\gamma} \equiv \frac{-\frac{\pi_i^*}{n} + (P - w^E - z + x_i^*) \frac{dq_i^*}{dn} + q_i^* \left( \frac{dp}{dn} - \frac{dw^E}{dn} + \frac{dx_i^*}{dn} \right)}{(n - 1)x_i^* \frac{dx_i^*}{dn} + (n - 1)q_i^* \frac{dx_i^*}{dn} + q_i^* x_i^*},
\]

and \((n - 1)x_i^* \frac{dq_i^*}{dn} + (n - 1)q_i^* \frac{dx_i^*}{dn} + q_i^* x_i^* > 0\).

**Proposition 2:** In the general demand case, when \( \gamma > \hat{\gamma} \) and the entry cost for the upstream firms is moderate, an increase on the number of downstream firms will increase the industry profit.

In the case without R&D, Eq. (8) can be rewritten as the following expression:

\[
\frac{d\pi_i^*}{dn} = \pi_i^* + n \frac{d\pi_i^*}{dn}.
\]

At upstream regulatory entry, we have that \( \frac{dw^*}{dn} = \frac{dw^E}{dn} > 0 \), and \( \frac{dp}{dn} - \frac{dw^E}{dn} < 0 \). Due to the presence of the business stealing effect, entries of downstream firms not only reduce the profit of individual firm but also reduce the total profit of industry. At upstream free-entry equilibrium, we have that \( \frac{dw^*}{dn} = \frac{dw^E}{dn} < 0 \), and the signs of \( \frac{dp}{dn} - \frac{dw^E}{dn} \) and \( \frac{d\pi_i^*}{dn} \) are ambiguous. There are two effects resulting from the entry of downstream firms. The first is the business-stealing effect, and the second one is input price-reducing effect.

Matsushima (2006) showed that when \( n = 2 \) or \( n = 3 \) in the linear demand case, the industry profits may be larger than those in the case of a downstream monopoly, is due to that
the input price-reducing effect is larger than the business-stealing effect. The industry profits of downstream firms competing in quantity may then increase with the number of downstream firms.

In our framework with R&D investment and knowledge spillover, there are three effects resulting from the entry of downstream firms and upstream competition. The first one is the business-stealing effect, the second one is input price-reducing effect, and the third one is knowledge-spillover effect. The business-stealing effect of upstream entry depends on knowledge-spillover effect is because upstream competition leads to higher levels of investment by the downstream firms. Hence, aggregate downstream profit may then highly be possible to increase with the number of downstream firms, when the knowledge-spillover effect is not too large.

In the following subsection, we confirm that this result holds for a linear demand case.

### 3.2 Linear demand case

The profit function of downstream firm is

\[
\pi_i = \left( a - Q - w - z + x_i + \gamma \sum_{d \neq i}^{n} x_d \right) q_i - x_i^2 / 2. \tag{9}
\]

The equilibrium outputs and R&D investments are

\[
q_i^* = \frac{(1+n)(a-z-w)}{B}, \quad x_i = \frac{2(a-z-w)(n(1-\gamma)+\gamma)}{B}, \quad i = 1 \ldots n, \tag{10}
\]

where \( B = 1 + n^2 - 2(n-1)^2 \gamma + 2(n-1)^2 \gamma^2 \). The second-order condition is

\[
\frac{2(n + \gamma - n\gamma)^2}{(1+n)^2} < 1, \quad \text{which guarantees that } B > 0, \text{ when } 0 \leq \gamma \leq 1/2. \quad \text{B > 0 is needed for finding a positive interior solution of the equilibrium R&D investment. However, if } \gamma > 1/2 \text{ there would have been no business stealing effect in the first place and upstream entry would have always increased downstream profit}. \tag{5}
\]

Letting \( Q_i = m q_i = n q_j \), the derived demand for the intermediate good follows from

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\(^5\) We appreciate the reviewer pointing out that the business stealing effect of upstream entry depends on knowledge spillover effect is because upstream competition leads to higher levels of investment by the downstream firms.
(10), and it is given by

\[ w = a - z - \frac{Bq_j}{n(n+1)} . \]  

(11)

The equilibrium output of the \( j \)th intermediate good producer is

\[ q_j^* = \frac{An(1+n)}{(1+m)B} . \]  

(12)

where \( A \equiv a - c - z \). The equilibrium price of intermediate good is computed as

\[ w^* = \frac{a-z+cm}{1+m} . \]  

(13)

which is irrelevant to the endogenous R&D of the downstream sector in the absence of upstream entry.

From Eq. (13), \( \frac{dw^*}{dm} = \frac{-A}{(1+m)^2} < 0 \): When the number of upstream firm increases, the input price \( w^* \) decreases. It confirms the result of Proposition 1 in the linear demand case.

We then want to find the free entry number of firms in the upstream sector considering R&D competition. The net profit of the \( j \)th intermediate good producer, \( j = 1, \ldots, m \) is

\[ \pi_j^* = \frac{A^2 n(n+1)}{(1+m)^2 B} - K. \]  

(14)

Free entry equilibrium number of intermediate good producers is given by \( \pi_j^* = 0 \), or

\[ \frac{A^2 n(n+1)}{(1+m)^2 B} = K. \]  

(15)

To illustrate how the endogenous variables are determined, Eq. (15) is setting to zero for free entry equilibrium and the endogenous number of upstream firms is derived as

\[ m^E = A \sqrt{\frac{(n+1)n}{KB}} - 1 \]  

(16)

where the superscript \( E \) denotes the free entry equilibrium.

The price of the intermediate good at free entry is computed as

\[ w^E = c + \frac{\sqrt{KB}}{\sqrt{n(n+1)}} . \]  

(17)

From Eq. (17),
10

\[
\frac{dw^E}{dn} = \frac{\sqrt{R}[2(n^2 + \gamma - n(n-2)\gamma + (n-1)(3n+1)\gamma^2) - (n+1)^2]}{2[n(n+1)]^{3/2} \sqrt{B}} < 0,
\]

when \( \frac{2(n+\gamma - n\gamma)^2}{(1+n)^2} < 1 \) and \( 0 \leq \gamma \leq \frac{1}{2} \). That is, when the number of downstream firms increases, the input price \( w^E \) decreases at upstream free-entry equilibrium in a linear demand case.

The above finding extends the result obtained in Matsushima (2006) in which R&D investment of the downstream firms is not considered.\(^6\)

Substituting Eqs. (10) and (16) into Eq. (9), and due to the complicated expression in (9), we consider the case of \( \gamma = 0 \) and \( \gamma = \frac{1}{2} \), and have

\[
\pi^*_i (\gamma = 0) = \frac{(1+2-n)n)(K+Kn^2+A^2n(1+n)-2A\sqrt{K}\sqrt{n(1+n)}\sqrt{1+n^2})}{n(1+n)(1+n^2)^2},
\]

\[
\pi^*_i (\gamma = \frac{1}{2}) = \frac{2A^2n+K(1+n)-2\sqrt{2A\sqrt{K}\sqrt{n(1+n)}}}{n(1+n)^2}.
\]

We now investigate how industry profits (of downstream firms) vary with the number of firms in the product market. The industry profits is

\[
\sum_{i=1}^{n} \pi^*_i (\gamma = 0) = \frac{(1+2-n)n)(K+Kn^2+A^2n(1+n)-2A\sqrt{K}\sqrt{n(1+n)}\sqrt{1+n^2})}{(1+n)(1+n^2)^2},
\]

\[
\sum_{i=1}^{n} \pi^*_i (\gamma = \frac{1}{2}) = \frac{2A^2n+K(1+n)-2\sqrt{2A\sqrt{K}\sqrt{n(1+n)}}}{n(1+n)^2}.
\]

We have following corollary immediately.

**Corollary 1:** When the demand function is linear, \( \gamma > \tilde{\gamma} \) and the entry cost for the upstream firms is moderate, an increase on the number of downstream firms will increase the industry profit.

**Proof:**

Differentiating Eqs. (20) and (21) with respect to \( n \), we obtain that

\(^6\) Matsushima and Mizuno (2012) demonstrated that under a simple Cournot model with linear demand and vertical relations, when downstream firms engage in process R&D, the profits of input suppliers for which upstream competition exists may be larger than those in which each input supplier has a bilateral monopoly relation with its buyer (downstream firm). It is because upstream competition leads to higher levels of investment by the downstream firms.
From Proposition 2 and Corollary 1, our argument of profit-raising entry holds for different demand function and critical value of $\gamma$. Matsushima (2006) shows that only when $n = 2$ or $n = 3$ in the linear demand case without downstream R&D, the industry profits may be larger than those in the case of a downstream monopoly. In this paper, entry may more likely be industry profit-raising when the entry cost for the upstream firms and the knowledge-spillover effect are moderate which can be seen from Proof (iii). There are three effects operating in product market: the first one is the business-stealing effect due to intensive competition from the product market; the second one is the input price-reducing effect, that comes from the lower input price with free entry of the upstream sector; and the third one is the knowledge-spillover effect that comes from the R&D investment of the downstream sector with knowledge spillover.

The synergy of knowledge-spillover and input price-reducing effects dominating the business-stealing effect is the reasoning for sustaining the non-falling industry profit in the downstream sector with endogenous technology choice.

4. Concluding Remarks

In this paper, we showed that an increase on the number of downstream firms might more likely be industry profit-raising, when the entry cost for the upstream firms and the knowledge-spillover effect are moderate. The input price-reducing effect associated with the knowledge-spillover effect dominating the business-stealing effect is the economic reason for
sustaining the non-falling industry profit in the downstream sector.
References


Appendix

Proof of $\frac{dq_i^*}{dx_i} > 0$, $\frac{dq_i^*}{dn} < 0$, and $\frac{dq_i^*}{dw} < 0$.

The first order conditions for profit maximization are

$$\frac{\partial \pi_i}{\partial q_i} = P - w - z + x_i + \gamma \sum_{d=1}^{n} x_d + P' q_i = 0. \quad (2)$$

From Eq. (2), we have

$$\frac{\partial^2 \pi_i}{\partial q_i \partial n} < 0, \quad \frac{\partial^2 \pi_i}{\partial q_i \partial x_i} > 0, \quad \text{and} \quad \frac{\partial^2 \pi_i}{\partial q_i \partial w} < 0.$$

Taking total differentiation to Eq. (2) and rearranging the terms yields

$$\frac{\partial^2 \pi_i}{\partial q_i} \frac{dq_i}{\partial n} = -\frac{\partial^2 \pi_i}{\partial q_i \partial x_i} \frac{dx_i}{\partial n} - \frac{\partial^2 \pi_i}{\partial q_i \partial w} \frac{dw}{\partial n}.$$

We obtain

$$\frac{dq_i^*}{dn} = -\frac{\partial^2 \pi_i}{\partial q_i \partial n} \frac{dq_i}{\partial q_i} < 0, \quad \frac{dq_i^*}{dx_i} = -\frac{\partial^2 \pi_i}{\partial q_i \partial x_i} \frac{dq_i}{\partial x_i} > 0, \quad \text{and} \quad \frac{dq_i^*}{dw} = -\frac{\partial^2 \pi_i}{\partial q_i \partial w} \frac{dq_i}{\partial w} < 0.$$

Proof of $\frac{dx_i^*}{dn} < 0$.

The first order conditions for profit maximization are

$$\frac{\partial \pi_i}{\partial x_i} = (P' \frac{dq_i}{dx_i} + 1)q_i - x_i = 0. \quad (3)$$

From Eq. (3), we have

$$\frac{\partial^2 \pi_i}{\partial x_i \partial n} < 0, \quad \frac{\partial^2 \pi_i}{\partial x_i \partial x_i} < 0.$$

Taking total differentiation to Eq. (3) and rearranging the terms yields

$$\frac{\partial^2 \pi_i}{\partial x_i} \frac{dx_i^*}{\partial x_i} = -\frac{\partial^2 \pi_i}{\partial x_i \partial n} \frac{dx_i^*}{\partial n}.$$

$$\frac{dx_i^*}{dn} = -\frac{\partial^2 \pi_i}{\partial x_i \partial x_i} \frac{dx_i^*}{\partial x_i} < 0.$$