Case 8.1 Moving Sand at the Brisbane Airport

When the Brisbane International Airport was redeveloped, about 2 million cubic meter of sand dredged from a nearby bay had to be moved by pipeline to various sites. Additional sand was brought to help compress the swampy grounds at the sites, and the excess sand was transported from these areas to other places around the airport by truck or scraper. The distance from each source to each destination, as well as the quantity of sand available from each source and the quantity required at each destination, is shown below. The use of linear programming in the case resulted in a saving in transport costs of about $400,000.

Source	Distance (meters) from source to destination:									Quantity available (cubic meters)
	Local-izer N	Exten-sion	Low areas	Roads	Car park	Local-izer S	Fire station	Oil Industry	Perimeter Road	
Apron	22	26	12	10	18	18	11	8.5	20	***960***
Terminal	20	28	14	12	20	20	13	10	22	***201***
Airline area	16	20	26	20	1.5	28	6	22	18	***71***
Maintenance	20	22	26	22	6		2	21	18	***24***
Access Road	22	26	10	4	16		24	14	21	***99***
Quantity required (cubic meters)	***62***	***217***	***444***	***315***	***50***	***7***	***20***	***90***	***150***	***1,355***

(a) If the cost of hauling sand is proportional to the amount of sand times the distance hauled, what is an expression for the quantity that the airport executives should maximize or minimize?

(b) What constraints must be satisfied?

Solution:

(a) Minimize $\sum_i \sum_j T_{ij} D_{ij}$ where T_{ij} is the number of cubic meters of sand to be moved from source *i* to destination *j*, and D_{ij} is the distance (in meters) from source *i* to destination *j*. This is the sum of the amount of sand times the distance hauled. It is proportional to the total cost of hauling the sand. The values of D_{ij} are given. The problem is to find the values of T_{ij} that minimize $\sum_i \sum_j T_{ij} D_{ij}$.

(b) The amount of sand moved from the *i-th* source must not exceed the amount available; that is, $\sum_j T_{ij} \le K_i$, where K_i is the quantity available of the *i-th* source. The amount of sand moved to the *j-th* destination must not be less than the amount required; that is That is, $\sum_i T_{ij} \ge R_j$, where R_j is the required total amount to the *j-th* destination. For all *i* and *j*, T_{ij} must be nonnegative; that is, $T_{ij} \ge 0$.

Case 8.2 How Linear Programming Improves Aircraft Operations

Linear Programming is used in a wide variety of industries and firms. Consider, for example, the case of Delta Airlines, which uses linear programming to reduce the costs of moving flight crews from place to place. Because of work rules limiting the number of hours per day that crew members can fly, as well as waiting time in airports and other factors, a substantial portion of the time of pilots and flight attendants is essentially unproductive, from the point of view of the airline.

In keeping with Delta's ongoing efforts to reduce unproductive time and costs, attention was focused on the problem of scheduling crews (8,500 pilots and 17,600 flight attendants) to cover approximately 4,900 flight segments daily to more than 220 cities worldwide; using more than 550 aircraft of eight types flown from a dozen different crew bases. To help solve this very large and complex problem, Delta introduced a new software module, developed in-house. This linear programming software uses interior point methodology. Additionally, it provides the optimal value without having to generate the billions of possibilities; this makes it extremely useful in crew scheduling to analyze different schedules and changes in work rules.

The results of this new module have provided Delta with an estimated annual saving of about $20 million. In addition to the monetary savings, there are improvements in quality-of-life issues since crew members now have schedules that take them away from their home bases for less time than before. Along with scheduling crews, Delta is currently using linear programming to help address a variety of planning issues, including maintenance of aircraft, fleet assignment, and labor planning.

Government agencies, like firms, use linear programming to help solve a variety of problems concerning aircraft operations. For example, the Air Force Military Aircraft Command has used linear programming to help perform aircraft operations more efficiently. In some problems involving the scheduling of military support aircraft, over 300,000 variables and almost 15,000 constraints were considered. While problems of this sort are far more complex than those considered in this chapter, the basic concepts are no different from those taken up here.

Case 8.3 How H. J. Heinz Minimize Its Shipping

The H. J. Heinz Company manufactures ketchup in a variety of factories scattered around the United States and distributes it to warehouses that are also scattered around the country. To determine how much ketchup each factory should send to each warehouse, Heinz has used linear programming techniques. The capacities of each factory, the requirements of each warehouse, and freight rates are given below:

	Freight rates (cents per cwt.) from factory:												**Daily require-ments (cwt.)**
Ware-house	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	**XI**	**XII**	
A	16	16	6	13	24	13	6	31	37	34	37	40	***1,820***
B	20	18	8	10	22	11	8	29	33	25	35	38	***1,530***
C	30	23	8	9	14	7	9	22	29	20	38	35	***2,360***
D	10	15	10	8	10	15	13	19	19	15	28	34	***100***
E	31	23	16	10	10	16	20	14	17	17	25	28	***280***
F	24	14	19	13	13	14	18	9	14	13	29	25	***730***
G	27	23	7	11	23	8	16	6	10	11	16	28	***940***
H	34	25	15	4	27	15	11	9	16	17	13	16	***1,130***
J	38	29	17	11	16	27	17	19	8	18	19	11	***4,150***
K	42	43	21	22	16	10	21	18	24	16	17	15	***3,700***
L	44	49	25	23	18	6	13	19	15	12	10	13	***2,560***
M	49	40	29	21	10	15	14	21	12	29	14	20	***1,710***
N	56	58	36	37	6	25	8	19	9	21	15	26	***580***
P	59	57	44	33	5	21	6	10	8	33	15	18	***30***
Q	68	54	40	38	8	24	7	19	10	23	23	23	***2,840***
R	66	71	47	43	16	33	12	26	19	20	25	31	***1,510***
S	72	58	50	51	20	42	22	16	15	13	20	21	***970***
T	74	54	57	55	26	53	26	19	14	7	15	6	***5,110***
U	71	75	57	60	30	44	30	30	41	8	23	37	***3,540***
Y	73	72	63	56	37	49	40	31	31	10	8	25	***4,410***
Daily capacity	***10,000***	***9,000***	***3,000***	***2,700***	***500***	***1,200***	***700***	***300***	***500***	***1,200***	***2,000***	***8,900***	***40,00***

The optimal daily shipment from each factory to each warehouse is shown in the table below. For example, all warehouse A's ketchup should come from factory I.

(a) According to officials of the H. J. Heinz Company, one of the most important advantage gained from the introduction of linear programming was that senior members of the distribution department no longer had to spend so much time preparing shipping programs. Who did it instead?

(b) They also said that an important advantage of linear programming was the peace of mind resulting from being sure that the program is the lowest-cost program possible. Does this mean that if the data on freight rates are incorrect, the program will still be optimal?

(c) What is the H. J. Heinz Company trying to minimize?

(d) Explain in detail the nature of the constraints.

	Factory												
Ware-house	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	**XI**	**XII**	**Total**
A	1,820												***1,820***
B	1,530												***1,530***
C		2,360											***2,360***
D	100												***100***
E		280											***280***
F		730											***730***
G	940												***940***
H				1,130									***1,130***
J		4,150											***4,150***
K	700		3,000										***3,700***
L	1,360					1,200							***2,560***
M		140		1,570									***1,710***
N	580												***580***
P								30					***30***
Q		1,340			500				500			500	***2,840***
R	810						700						***1,510***
S								90				880	***970***
T												5,110	***5,110***
U	2,160							180		1,200			***3,540***
Y											2,000	2,410	***4,410***
Total	***10,000***	***9,000***	***3,000***	***2,700***	***500***	***1,200***	***700***	***300***	***500***	***1,200***	***2,000***	***8,900***	***40,000***

(a) A computer, together with some lower-level people.

(b) No. If the data concerning freight rates, daily requirements, or daily capacities are wrong, the results obviously may wrong as well.

(c) Heinz is trying to minimize its total freight bill, which can be represented as $\sum_i \sum_j U_{ij} V_{ij}$, where U_{ij} equals the freight rate from factory *i* to warehouse *j*, and V_{ij} equals the amount shipped per day from factory *i* to warehouse *j*.

(d) One set of constraints says that the total shipments from each factory cannot exceed its capacity. That is, $\sum_j V_{ij} \leq K_i$, where K_i is the capacity of the *i-th* factory. Another set of constraints says that the total shipments to each warehouse must meet its requirements. That is, $\sum_i V_{ij} \geq R_j$, where R_j is the required total shipment to the *j-th* warehouse. In addition, there are non-negativity constraints saying that the shipment from each factory to each warehouse must be non-negative. That is, $V_{ij} \geq 0$.

Case 8.4 Evaluating the Organization of a Shipping Program

A leading manufacturer and distributor of a consumer good was organized around the principle of decentralized management. In accord with this principle, each of the firm's six regional warehouses was under the supervision of a regional sales manager, who decided how much of the firm's product to order from each of the firm's plants. Because the regional warehouse had to pay the freight costs, it was felt that each regional sales manager would formulate its orders so as to minimize both its own freight costs and those of the firm as a whole.

But one of the firm's plants was located far from all the firm's warehouses, and none of the regional sales managers willingly ordered from it. Only when the other plants turned down orders, pleading lack of capacity to fulfill them, did the regional sales managers order from this distant plant. In general, the other plants accepted orders on a first-come, first-served basis, and turned down those that were received last.

If you were a consultant to this firm, what suggestions would you make concerning the organization of the firm's shipping program?

Answer:

It is very unlikely that the firm is minimizing its total cost freight costs. Each regional sales manager cannot see all the interrelated aspects of the problem, and each is concerned with minimizing only his or her own freight costs. Moreover, the choice of which orders are fulfilled by the distant plant is being made on the basis of which orders are received (by the order plants), not with an eye toward minimizing total freight costs. To see how much those costs could be reduced, the firm might use the sort of linear programming model. This model should be used to determine how much each plant should ship to each warehouse to minimize total freight costs.