Quantitative Method
Assignment 2
Due October 18, 2005

1. Find the eigenvalues and eigenvectors of the matrix $\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$.

   (a) Verify that two eigenvectors are orthogonal to each other. Is it always true?

   (b) Is the determinant of **A** equal to the product of all eigenvalues? Why?

   (c) What is the rank of **A**? Explain.

   (d) Is the matrix **A** positive definite, positive semi-definite, negative definite, or negative semi-definite? Why?

   (e) Can we find $\mathbf{A}^{\frac{1}{2}}$ such that $\mathbf{A}^{\frac{1}{2}}\mathbf{A}^{\frac{1}{2}} = \mathbf{A}$? Why?

   (f) Does $\mathbf{A}^{-1}$ exist? If it exists, find its eigenvalues and eigenvectors.

2. Using the characteristic equation, show for λ being an eigenvalue of **A** that $1/(1+\lambda)$ is an eignevalue of $(\mathbf{I}+\mathbf{A})^{-1}$.

3. When the (symmetric) matrix of **A** is positive definite, prove that eigenvalues of $(\mathbf{A}+\mathbf{A}^{-1})$ are equal or greater than 2.

4. Show that a symmetric idempotent matrix $\mathbf{A}_{n\times n}$ can be diagonalized as:

$$\mathbf{C}'\mathbf{A}\mathbf{C} = \begin{bmatrix} \mathbf{I}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \text{ where } \mathbf{C}'\mathbf{C} = \mathbf{I}_n \text{ and trace}(\mathbf{A}) = r.$$

5. Are the following quadratic forms positive for all values of $\underline{\boldsymbol{x}}$?

   (a) $y = x_1^2 - 28x_1x_2 + 11x_2^2$.

   (b) $y = 5x_1^2 + x_2^2 + 7x_3^2 + 4x_1x_2 + 6x_1x_3 + 8x_2x_3$.