Quantitative Method

Assignment 3

Due October 25, 2005

1. Find the eigenvalues and eigenvectors of the matrix $\mathbf{A} = \begin{bmatrix} 5 & 1 \\ 2 & 4 \end{bmatrix}$. Are two eigenvectors orthogonal to each other? Why?

2. Let $x_1, x_2, \ldots, x_n \overset{iid}{\sim} N(\mu, \sigma^2)$;

   i.e., $\mathbf{x} \sim N\left(\mu \underline{\mathbf{1}}, \sigma^2 \mathbf{I}_n\right)$ where $\mathbf{x} = \left(x_1, x_2, \ldots, x_n\right)' \in \Re^n$ and $\mathbf{1} = \left(1,1,\ldots,1\right)' \in \Re^n$.

   (a) Show that

   $$\frac{(n-1)S^2}{\sigma^2} = \frac{\sum_{i=}^{n}\left(x_i - \bar{x}\right)^2}{\sigma^2} \sim \chi^2_{(n-1)} .$$

   (b) From the definition of t distribution (note that the random variable on the numerator is independent of that on the denominator) show why

   $$\frac{\bar{x} - \mu}{S/\sqrt{n}} \sim t_{(n-1)} .$$

   (Hint: $\left(1/\sigma\right)\left(x - \mu\right) \sim MN\left(\underline{\mathbf{0}}, \mathbf{I}_n\right)$.)

3. Show that eigenvalues of an idempotent matrix (may not be symmetric) are all 0 and 1.

4. Give an example that a matrix with eigenvalues of 0 and 1 is not idempotent.

5. Prove that a symmetric matrix is idempotent if and only if its eigenvalues are all zero and unity.