

Quantitative Method

Assignment 5

Due November 8, 2005

1. Given the following two primals, formulate their duals:

$\begin{aligned} & \max_{x_1, x_2} && \pi = 9x_1 + x_2 \\ \text{(a) subject to} &&& 2x_1 + x_2 \leq 8 \\ &&& 4x_1 + 3x_2 \leq 14 \\ &&& x_1, x_2 \geq 0 \end{aligned}$	$\begin{aligned} & \min_{x_1, x_2} && C = x_1 + 7x_2 \\ \text{(b) subject to} &&& \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} 5 \\ 4 \\ 9 \end{bmatrix} \\ &&& x_1, x_2 \geq 0 \end{aligned}$
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2. A local business firm is planning to advertise a special anniversary sale on radio and television during a particular week. For that, a maximum budget of \$16,000 is approved. It is found that radio commercial cost \$800 per 30-second spot (call this x_1), with a minimum contract of five spots. Television commercials, on the other hand, cost \$4,000 per spot (call this x_2). Because of heavy demand, however, only four television spots are still available in the designated week. On the basis of estimated size of audience and other factors, it is believed that a television spot is six times as effective as a radio spot in reaching potential customers.

- (a) Formulate a linear programming (LP) to allocate advertises that will attract the largest possible number of potential customers.
 - (b) Solve the LP.
 - (c) Formulate the dual problem.
 - (d) Find and interpret the shadow prices for this LP.
 - (e) How will the optimal solution be affected if the availability of television commercial spots is not constrained?
3. My diet comes from the following four kinds of food consumption: brownies, chocolate ice cream, cola, and pineapple cheesecake. Each brownie costs 50¢, each scoop of chocolate ice cream costs 20¢, each bottle of cola costs 30¢, and each piece of pineapple cheesecake costs 80¢. Each day, I must ingest at least 500 calories, 6 oz of chocolate, 10 oz of sugar, and 9 oz of fat. The nutrition content per unit of each food is shown in the following table:

Type of food	Calories	Chocolate (oz)	Sugar (oz)	Fat (oz)
Brownie	400	3	2	2
Chocolate ice cream (1 scoop)	200	2	2	4
Cola (1 bottle)	150	0	4	1
Pineapple cheesecake (1 piece)	500	0	4	5

- (a) Formulate and solve a linear programming (LP) that can be used to satisfy my daily nutritional requirement at minimum cost.
- (b) Find and interpret the shadow prices for this LP.