Quantitative Method

Assignment 8

Due December 20, 2005

1. A dependent variable **Y** (20 × 1) was regressed onto three independent variables plus an intercept (so that **X** was of dimension 20 × 4). The following matrices were computed:

$$\mathbf{X'X} = \begin{bmatrix} 20 & 0 & 0 & 0 \\ 0 & 250 & 401 & 0 \\ 0 & 401 & 1{,}013 & 0 \\ 0 & 0 & 0 & 128 \end{bmatrix} \qquad \mathbf{X'Y} = \begin{bmatrix} 1{,}900.00 \\ 970.45 \\ 1{,}674.41 \\ -396.80 \end{bmatrix} \qquad \mathbf{Y'Y} = 185{,}883.$$

(a) Compute the OLS estimate vector $\hat{\boldsymbol{\beta}}' = (\hat{\beta}_0 \quad \hat{\beta}_1 \quad \hat{\beta}_2 \quad \hat{\beta}_3)$ and write the regression equation.

(b) Construct the ANOVA table.

(c) Compute the estimate of σ^2 and the standard error for each regression coefficient. Compute $\mathrm{Cov}(\hat{\beta}_1, \hat{\beta}_2)$ and $\mathrm{Cov}(\hat{\beta}_1, \hat{\beta}_3)$.

(d) Drop the last **X,** $\boldsymbol{x}_3$, from the model. Reconstruct $\mathbf{X'X}$ and $\mathbf{X'Y}$ for this model without $\boldsymbol{x}_3$ and repeat questions (a) and (b). Put $\boldsymbol{x}_3$ back in the model but drop the next to last **X,** $\boldsymbol{x}_2$, and repeat questions (a) and (b).

   (i) Which of the independent variables, $\boldsymbol{x}_2$ or $\boldsymbol{x}_2$, made the greater contribution to Y in the presence of the remaining **X**'s?

   (ii) Explain why $\hat{\beta}_1$ changed in value when $\boldsymbol{x}_2$ dropped but not when $\boldsymbol{x}_3$ was dropped.

(e) From the inspection of $\mathbf{X'X}$, how can you tell that $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, and $\boldsymbol{x}_3$ were expressed as deviation from their respective means? Would $(\mathbf{X'X})^{-1}$ have been easier or harder to obtain if the original **X**'s (without subtraction of their means) had been used? Explain.

2. Data on the aggregate production for the U.S. manufacturing sector are contained in the file *manuf.xls*. The variables are

Y: The gross output

K: Capital

L: Labor

E: Energy

M: Other intermediate materials.

(a) Estimate, by least squares, the log-log model

$$\ln(Y) = \beta_1 + \beta_2 \ln(K) + \beta_3 \ln(L) + \beta_4 \ln(E) + \beta_5 \ln(M) + \varepsilon$$

(b) *Discuss* and *interpret* the estimates of β_2, β_3, β_4 and β_5.

(c) Verify the coefficients $\beta_2, \beta_3, \beta_4, \beta_5$ are ***partial coefficients***.

(d) Use the variance inflation factor (VIF) to examine the presence of multicollinearity.

(e) Are the **signs** and **relative** magnitudes of the estimates of β_2 and β_3 consistent with economic logic?