Quantitative Method

Assignment 1

Due October 3, 2006

1. Consider $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

   (a) Compute **ABC**, **CAB**, **BCA**, and $\mathbf{CB}'\mathbf{A}'$.

   (b) Verify that $(\mathbf{ABC})' = \mathbf{C}'\mathbf{B}'\mathbf{A}'$.

2. Evaluate the determinant of $\mathbf{D} = \begin{bmatrix} 1 & 6 & 4 & 3 \\ 2 & 8 & 5 & 4 \\ 3 & 8 & 7 & 5 \\ 4 & 9 & 7 & 7 \end{bmatrix}$.

3. Calculate $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$ for $\mathbf{A} = \begin{bmatrix} -1 & 3 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 3 \\ 0 & 0 & 7 & 0 \\ 0 & 4 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{bmatrix}$.

4. Assume that $\mathbf{A} = [a_{ij}]_n$ is a triangular matrix. Show that $\det(\mathbf{A}) = \prod_{k=1}^{n} a_{kk}$.

5. Assume that a square matrix **A** with order n is invertible.

   (a) Show that $(\operatorname{adj}(\mathbf{A}))^{-1} = \operatorname{adj}(\mathbf{A}^{-1})$.

   (b) Show that $\det(\operatorname{adj}(\mathbf{A})) = (\det(\mathbf{A}))^{n-1}$.