Quantitative Method

Assignment 2

Due October 18, 2006

1. Consider $\boldsymbol{x}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\boldsymbol{x}_2 = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$, $\boldsymbol{x}_3 = \begin{bmatrix} -13 \\ -1 \\ 2 \end{bmatrix}$, and $\boldsymbol{x}_4 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

   (a) Are $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3$ linearly independent? If they are not, find a linear relationship among them.

   (b) Are $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_4$ linearly independent? If they are not, find a linear relationship among them.

2. Let the column vector $\mathbf{1}_n' = (1,1,\ldots,1) \in \Re^n$.

   (a) Show $\text{rank}\left(\mathbf{1}_n(\mathbf{1}_n'\mathbf{1}_n)^{-1}\mathbf{1}_n'\right) = 1$.

   (b) Verify $\mathbf{M}^0\mathbf{M}^0 = \mathbf{M}^0$, where $\mathbf{M}^0 = \mathbf{I} - \mathbf{1}_n(\mathbf{1}_n'\mathbf{1}_n)^{-1}\mathbf{1}_n'$.

3. Show that $\text{tr}(\mathbf{A}+\mathbf{B}) = \text{tr}(\mathbf{A}) + \text{tr}(\mathbf{B})$ and $\text{tr}(a\mathbf{A}) = a \cdot \text{tr}(\mathbf{A})$.

4. Find the inverse of $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & -1 \\ 0 & 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$ if it exists.

5. Find the eigenvalues of the following matrices.

   (a) $\mathbf{A} = \begin{bmatrix} 5 & 1 \\ 2 & 4 \end{bmatrix}$

   (b) $\mathbf{B} = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$