Quantitative Method
Assignment 3
Due October 31, 2006

1. Consider the matrix $\mathbf{A} = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$.

    (a)   Find the eigenvalues and the corresponding eigenvectors

    (b)   What is the rank of $\mathbf{A}$?   Explain.

    (c)   Does $\mathbf{A}^{-1}$ exist?   If it exists, find its eigenvalues and eigenvectors.

    (d)   Find the determinant of $\mathbf{A}^{20}$.

    (e)   Find the trace of $\mathbf{A}^{25}$.

    (f)   Find the trace of $2\mathbf{A}^3 + 3\mathbf{A}^2 + 7\mathbf{A} + 5\mathbf{I}$.

2. Consider the matrix $\mathbf{D} = \begin{bmatrix} 5 & 1 \\ 2 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$.

    (a)   Find their eigenvalues and the corresponding eigenvectors.

    (b)   Are the eigenvectors of $\mathbf{D}$ orthogonal to each other?   Why?

    (c)   Are the eigenvectors of $\mathbf{B}$ orthogonal to each other?   Why?

    (d)   Can we find $\mathbf{B}^{\frac{1}{2}}$ such that $\mathbf{B}^{\frac{1}{2}} \mathbf{B}^{\frac{1}{2}} = \mathbf{B}$?   Why?