Quantitative Method
Assignment 5
Due November 14, 2006

1. Assume $\mathbf{y}_{n\times 1} \sim MN(\mathbf{0}, \mathbf{I}_n)$, and $\mathbf{A}$ is symmetric and idempotent.
   (a) Show that if $\mathbf{L}$ is an $m \times n$ non-random matrix, then $\mathbf{Ly}$ and $\mathbf{y}'\mathbf{Ay}$ are <u>independent</u> if $\mathbf{LA} = \mathbf{0}$.
   (b) Show that if $\mathbf{b}$ is an $n \times 1$ non-random vector and $\mathbf{Ab} = \mathbf{0}$,

   then $\dfrac{\mathbf{b}'\mathbf{y}/\sqrt{\mathbf{b}'\mathbf{b}}}{\sqrt{\mathbf{y}'\mathbf{Ay}/q}} \sim t_q$ where $q = \text{tr}(\mathbf{A})$.

   (c) $\dfrac{(1/\sqrt{n})\,\mathbf{1}'\mathbf{y}}{\sqrt{\mathbf{y}'\mathbf{M}^0\mathbf{y}/(n-1)}} \sim t_{(n-1)}$.

2. Let $x_1, x_2, \ldots, x_n \overset{iid}{\sim} N(\mu, \sigma^2)$; i.e., $\mathbf{x} \sim MN(\mu\underline{\mathbf{1}}, \sigma^2\mathbf{I}_n)$ where $\mathbf{x} = (x_1, x_2, \ldots, x_n)' \in \Re^n$ and $\mathbf{1} = (1,1,\ldots,1)' \in \Re^n$.

   (a) Show that

$$\frac{(n-1)S^2}{\sigma^2} = \frac{\sum_{i=}^{n}(x_i - \bar{x})^2}{\sigma^2} \sim \chi^2_{(n-1)}.$$

   (b) From the definition of t distribution (note that the random variable on the numerator is independent of that on the denominator) show why

$$\frac{\bar{x} - \mu}{S/\sqrt{n}} \sim t_{(n-1)}.$$

   (Hint: $(1/\sigma)(x\mathbf{1} - \mu\mathbf{1}) \sim MN(\underline{\mathbf{0}}, \mathbf{I}_n)$)

3. Assume $\mathbf{y}_{n\times 1} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Show that $E(\mathbf{y}'\mathbf{A}\mathbf{y}) = \text{tr}(\mathbf{A}\boldsymbol{\Sigma}) + \boldsymbol{\mu}'\mathbf{A}\boldsymbol{\mu}$.

   (Hint: $E(\mathbf{y}'\mathbf{A}\mathbf{y}) = \text{tr}[E(\mathbf{y}'\mathbf{A}\mathbf{y})] = E[\text{tr}(\mathbf{y}'\mathbf{A}\mathbf{y})]$)